The Impact of Inequality on Asset Prices When Households Care About Wealth

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Abstract

This thesis explores whether wealth inequality could be the cause of recent unpredicted rises in asset prices and sustained falls in interest rates, by using fixed-factor, heterogeneous agent models with asset ownership appearing explicitly in the utility functions of agents. When inequality is very high, increasing inequality is shown to push asset prices up, and interest rates down. This finding holds for all levels of inequality when asset ownership appears in the utility functions of all agents in the same way. Finally, an overlapping generations model is used to show how high asset prices and low interest rates can be damaging to non-owners of assets within the framework of the model.

Preface

In 2008 global interest rates fell to historically low levels in all of the world's developed economies. Since then, financial markets and, where they make public predictions, central banks, have predicted a renormalisation of interest rates every single year from 2009 until today. The vast majority of these predictions have been incorrect, and the period there have been extremely rapid and unpredicted rises in a variety of global asset prices.

From 2008 to 2014 I was an interest rates trader for a large investment bank, and I closely observed this phenomenal consistency of misprediction. I developed the idea that traditional economics was failing to understand the effect of increases in wealth inequality on interest rates and asset prices and, by speculating that interest rates would remain depressed for a long period as a result of increasing wealth inequality, became the bank's globally most profitable trader in 2011. I used the money I was paid to buy real assets, which have since soared in value.

I believe that continued increases in wealth concentration will continue to cause asset prices to rise, relative to wages, and interest rates to stay at low levels, and I believe that this will continue to cause poor people problems in obtaining good quality housing and saving for retirements. In this thesis I am taking my first steps towards formalising these ideas.

Thank you for taking the time to read this thesis. I hope that these ideas can be as interesting and profitable for you as they have been for me.

Economics should be about people.

Contents

Chapter 1

Introduction

This thesis explores the possibility that wealth inequality can lead to increases in asset prices and asset price to wage ratios, and decreases in effective interest rates. It does this through the creation of static and dynamic heterogeneous agent models with variable inequality, in which there is a fixed, tradeable factor of production, and in which asset ownership appears explicitly in the utility functions of the rich.

Four versions of the model are included: first a static model with only fixed assets where only the rich care about asset ownership explicitly; second a static model with only fixed assets where both the rich and the poor care about asset ownership explicitly, and two dynamic models, in both of which only the rich care about asset ownership explicitly - one including both fixed and accumulable forms of productive assets, and one with only fixed assets, where the poor agents are two stage overlapping generation (OLG) agents maximising discounted lifetime consumption, and the rich are infinitely lived.

The most basic form of the model demonstrates that, when inequality is sufficienty low, asset price and asset price to wage ratios both decrease rapidly as equality increases, and effective interest rates rise. For lower levels of inequality, however, the effects becomes smaller and can potentially reverse in sign, depending upon chosen parameter values.

In the second form of the model, once both rich and poor agents are made to care explicitly about wealth ownership, which is done by giving them identical utility functions, the non-monoticity and parameter dependence of this relationship breaks down - asset price and asset price wage ratios are shown to be monotonically decreasing in equality, and interest rates monotonically increasing in equality.

The third form of the model shows that the results are not significantly changed by extending the model into the dynamic horizon and including both fixed and accumulable forms of productive asset, although this result depends on the specific way in which we adapt the utility function to the inclusion of two assets. The fourth model demonstrates clearly the welfare loss that higher asset prices and lower interest rates can have directly on people who are not born with assets, even if they care only about consumption.

1.1 Existence of economic phenomena

1.1.1 Sharp rises in asset prices

The UK mean average house price in the second quarter of 1992, according to the UK Office for National Staistics, was £70,000. Average recorded income of mortgage borrowers was £24,000. By the third quarter of 2017, average house prices had risen £303,000, and the average recorded income of mortgage borrowers £63,000. This represents an increase in the house price to income ratio from 2.92 to 4.81 (ONS, 2019).

The same source demonstrates an even more dramatic increase in this ratio for London over the same period, with house price to income ratios for that specific city rising from less than 3 in 1996, to over 10 by 2017.

This pattern is not limited only to the United Kingdom. Data (see fig 1.1) shows similar increases in average house price to income ratios in France, Spain, the Netherlands, Denmark and Norway, and increases even faster than those of the UK in Canada, Australia, Ireland and New Zealand (RBA, 2017). Whilst the US, as a whole, has been spared this rise so far, this is not true of many of its largest cities, with this "affordability ratio" measure having increased significantly in New York, San Francisco and Los Angeles. The trend has been even more dramatic in the largest cities of India, and China, with the average 1,000 square foot apartment in central Shanghai now being valued at approximately 50 times the average Shanghai income. Recent work by Knoll et al. (2017) clearly lays out a picutre of dramatic increases in house prices across a number of countries.

This pattern of asset prices increasing at rates far quicker than that of wages of consumer goods prices is not limited only to housing. Data from the Royal Institute of Chartered Surveyors suggest that the average price of an acre of farmland in the UK increased from less than £5,000 in 2007 to over £10,000 by 2016, a rate of price inflation of over 8%. Stock markets have grown at a similarly inflation

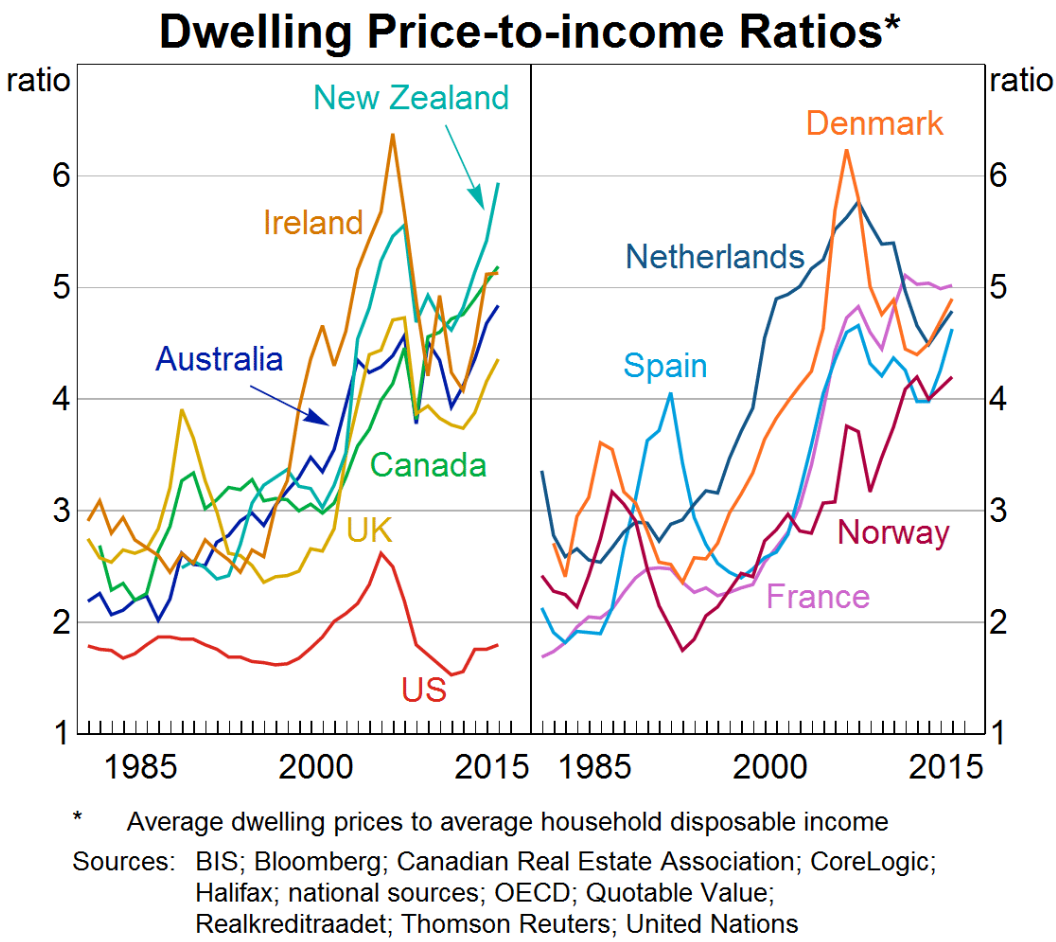


Figure 1.1: International movements in house price to income ratios

beating pace; even if growth is measured from the beginning of 2008, before the 2008 collapse, the US S&P 500 stock index has grown at a rate of over 6.5% per year, not including dividends. If we start measuring from the beginning of 2009, the ex-dividend growth is over 12% annually. Similar inflation beating growth can be seen in many international stock markets.

1.1.2 Deep and sustained falls in interest rates

Alongside these moves, across the same time periods, there has been a clear shift towards significantly lower interest rates, across a broad range of countries. This can be seen when looking at either central bank base rates, or the yields on longer term government bonds, and across every single developed market currency, with the single exception of Japan, which began the period already at the zero lower bound. In many countries, a variety of interest rates have become negative. The secular fall in interest across a broad range of countries can clearly be seen in fig 1.2, which shows short term interest rates across the last 17 years in all G7 countries.

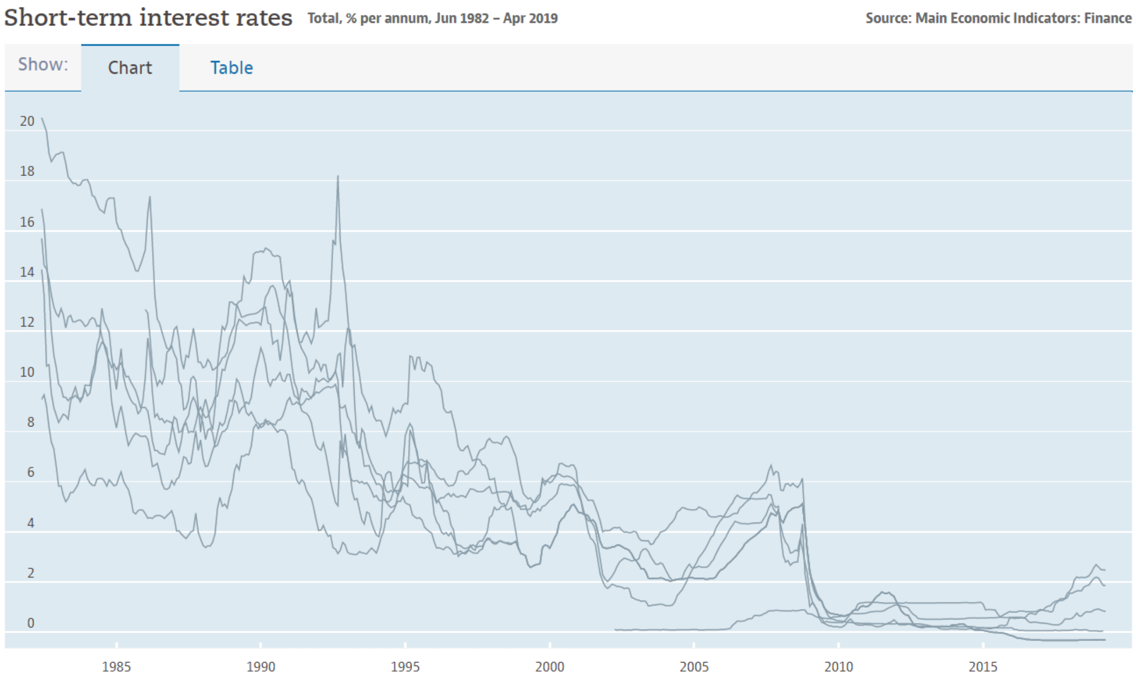


Figure 1.2: Short term interest rates for G7 countries 1982-2019 (OECD, 2019)

1.1.3 Potential relationship between asset prices and interest rates

These two phenomena may be related. A simple model of asset valuation as a discounted sum of a stream of future cashflows will show that asset prices are closely and inversely related to interest rates. It will also show that asset prices are particularly sensitive to changes in interest rates as the rates approach zero, which would explain well the particularly sharp rise in asset prices since 2008.

1.1.4 Increases in Wealth Inequality

At the same time as the aforementioned increases in asset prices and falls in interest rates, data from the World Inequality Report (Alvaredo et al., 2018) and the work of Thomas Piketty and others supports the idea that wealth inequality has risen quite sharply over the last 20 years. This thesis will discuss theoretically the possibility that the moves in asset prices and interest rates have been caused by the changes in wealth inequality. This contrasts with the majority of the recent wealth inequality literature, which focuses on the rises in wealth inequality itself and the reasons for it.

1.2 Argument that the phenomena is not well understood

1.2.1 Asset Prices

Whilst it can be understood that the dual phenomena of low interest rates and high asset prices are related, it is important to understand that the recent development of the two phenomena was not widely anticipated. The 6% annual increase in UK house prices over the period 1992-2017 with which I began this introduction would represent an approximately 11% annual return including rents, so significantly above the average interest rate of less than 4% during the period as to be difficult to reconcile through concepts of risk aversion alone. The 12% annual rise in US stocks (before dividends are included) during a period of zero interest rate policy can surely not be attributed to pure risk aversion. It must be accepted as having not been broadly anticipated.

1.2.2 Interest Rates

The unanticipated nature of the prolonged period of zero or near zero interest rates in the developed world is even clearer to see. Any analysis of historic predictions of future interest rates generated by markets will show that nearly all predictions made in advance of interest rates since the financial crisis have been significantly too high. This can clearly be seen in the below fig 1.3, created by Goldman Sachs Global Investment Research in 2016:

In this graph, the dark line shows the history of US central bank base rates before 2016. The many grey lines are predictions of the future base rate generated from market prices. Predictions have clearly been far too high, on average, especially in the period since the 2008 crisis.

The picture is even more startling if one looks at the predictions generated by central banks, where they exist. Despite central banks being the institutions that set interest rates, their predictions of interest rates, similarly to those of financial markets, have been consistently, significantly too high. To take just a single example, but one which is not at all atypical, we can look at the predictions provided by the governors of the US Fed in September 2014, at which time their base rate was approximately 0.12%. The average expectation was for rates to hit 1.27% at the end of 2015, 2.68% in 2016, and 3.54% in 2017. The actual path transpired to be 0.37%, 0.63% and 1.38% (US Federal Reserve, 2014). On average, the predicted

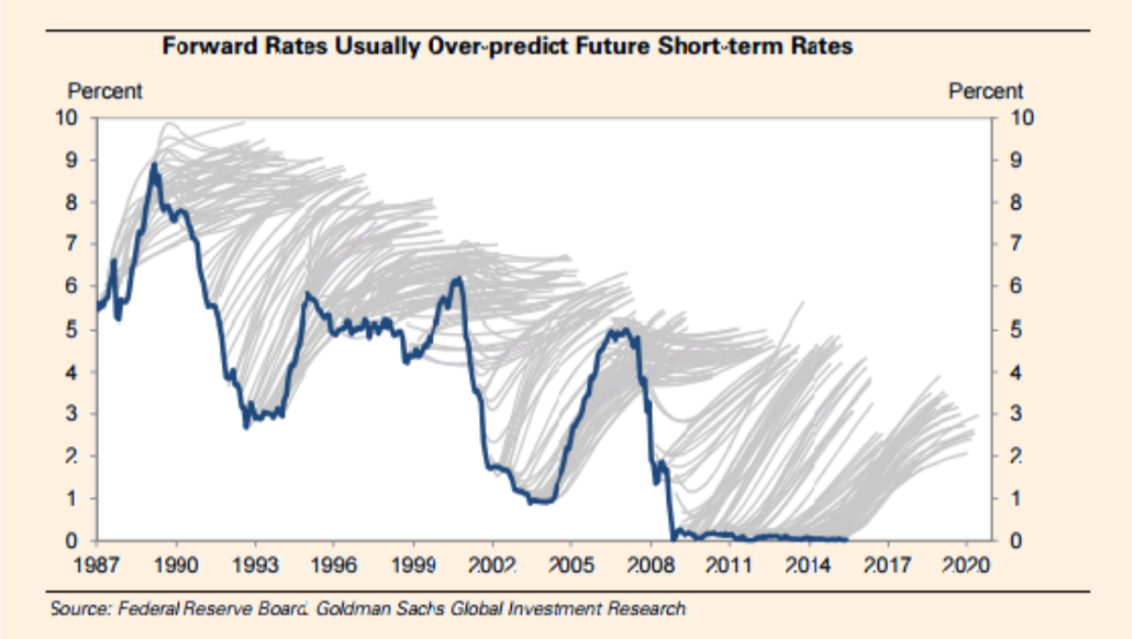


Figure 1.3: Overprediction of interest rates by markets. Source: Federal Reserve Board; Goldman Sachs Global Investment Research

number of hikes was just below four times the actual number of hikes delivered in each period. We can look at the individual predictions of each governor. Not a single governor underpredicted the path of rates for a single future data point. Every single governor was wrong by a large amount, in the same direction. The governors were unanimous in their incorrectness.

Whilst it could be argued (although I believe it would be charitable) that the central bank governors may have been being intentionally dishonest in their predictions, in an attempt to guide public expectations, the same cannot be said for market predictors, who are generally very well incentivised to predict honestly.

I believe that this evidence of a broad failure to predict both the prolonged low level of interest rates, and the extent of increases in asset prices is evidence that the causes of these phenomena are not understood. I thus believe that it is important to explore new ideas about the potential causes of these phenomena.

1.3 Importance of the phenomena

High levels of asset prices, relative to wages, make it more difficult for individuals born into poor families to accumulate wealth over their own lifetimes, and to leave wealth to their children.

Many individuals save wealth over the course of their lifetime in order to fund their retirement, and many pensioners rely on the capital income of their savings to support their retirement. An increased difficulty of accumulating assets when working could lead people born into poor backgrounds to be unable to afford comfortable retirements. An increased cost of housing, in particular, will clearly make it more difficult for individuals born without wealth to purchase a house from their employment income. Those that are able to buy a house, will have to take larger debts in order to do so. This means that higher asset prices mean people born without wealth will, across their lifetime, likely spend a greater amount of their income on rent or mortgage payments, which will probably negatively affect their quality of life. If high asset prices are combined with (or caused by) low general interest rates, even people who choose to save using assets other than housing may well find that they are unable to accumulate wealth, due to those assets being expensive and providing a low rate of compounding interest.

High asset prices also have effects upon inequality and social mobility. All other things being equal, asset price rises clearly benefit the rich, who own the assets, relative to the poor, who do not. If asset price rises are not matched by wage rises, it clearly becomes more difficult for those born without wealth to accumulate wealth. Thus high asset prices relative to wages mean that individuals at the bottom of the wealth distribution will find it hard to rise in the wealth distribution, meaning that high asset prices relative to wages are bad for social mobility. If asset prices continue to rise relative to wages, it can be expected that the wealth distribution will lose connection to work and wage income over time, and will start to reflect largely historic, dynastic effects. These effects could be compounded if the capital share of income rises, which it appears to have been doing in recent years (OECD, 2012).

The "unaffordability" of housing has become a growing concern for a great number of people in a wide variety of cities and countries across the world, as represented by its increasing salience in global politics and media. This can be seen in recent proposals by the UK Labour party that the Bank of England should specifically target limiting house price inflation (The Guardian, 2019).

1.4 Structure of the thesis

After a review of relevant literature in chapter 2, I will begin chapter 3 with a brief discussion of the assumptions and new concepts that I will be using in the models. I then, in chapter 4, introduce and solve the model in its simplest form - a static model with a single, fixed productive asset. This produces some results regarding the effect of inequality on asset price to wage ratios and interest rates as well as the effect of technological change. I then generalise this model to attempt to explore these results.

In chapter 5 I relax a simplifying assumption made about the behaviour of the poor - allowing both the poor and the rich to have increasing propensity to save as their income increases. Under this extension, it is shown that asset price/wage ratios are monotonically decreasing as the equality parameter is increased.

In chapter 6, I extend the model into a dynamic setting, where there are two forms of productive asset, one which is fixed and one which can be created - often described as "land" and "capital". This shows that, at the steady state of the dynamic model, the relationship between inequality, asset affordability, and interest rates is not fundamentally changed, but that this depends on the specific way in which the utility functions of the rich are extended to incorporate the increased number of assets.

In chapter 7, I revert to the single asset model, but remain in a dynamic framework, this time making the poor OLG agents. This is to enable me to present the way in which expensive assets can directly affect the consumption of the poor through the channel of affecting their ability to save for retirement. This model shows that, even when it does not affect the asset holdings of the poor directly in any way, increased inequality can have a direct negative effect on the consumption of the poor, by leading them to have a lower capital share of income in their retirement. Finally, in chapter 8, I outline the results in their totality and provide some discussion.

Chapter 2

Literature Review

2.1 General Equlibrium

The theoretical models in my paper are all general equilibrium models, an extremely well established field of economic models which dates back over one hundred years to the work of Leon Walras, and was developed greatly in the 1950's by Kenneth Arrow, Gérard Debreu, and Lionel W. McKenzie (Arrow and Debreu, 1954). The technique is widely used and taught today at undergraduate degrees across the world, and will be very familiar to anyone familiar with modern economics writings.

2.1.1 General Equilibrium with Fixed Factors

All of the models in this paper will involve a fixed factor, sometimes as a unique productive factor alongside labour, and at other times alongside a third productive factor which can be created and accumulated. Whilst it is currently popular to create economic models which do not include a fixed factor, there is a strong tradition of fixed factor models, often referring to the fixed factor as "land" and the accumulable factor as "capital", going back at least as far as Feldstein (1977). Within these models there is often a tradition of discriminating between the rental incomes of "capital" and "land" by referring to the first as "interest" and the second as "rent".

In Feldstein's theory paper of 1977 such a model is used to demonstrate the potentially complicated incidence of a tax on land. He creates an overlapping generations model within which a tax on land, rather than being completely neutral as had previously been theorised, incentivises asset holders to increase capital accumulation. This, in turn, increases the steady state capital level and, under typical assumptions on production functions, increases the marginal product of land, and hence both the rental income and price of land. Overall effects of land taxes on the price of land are ambiguous in the model, against intuition and previous results that land taxes would push land prices unambiguously down.

Feldstein's paper is itself a critique of earlier work on fixed factor models concerning the taxation of land, and he mentions Ricardo (1951), Pigou (1947) and Dalton (1954) as all concerning models with land.

Whilst models concerning fixed factors are in the minority in recent work, there is some modern interest in them, for example Stiglitz (2015) who discusses the propensity for the land share of income to increase over time.

2.1.2 General equilibrium "class models"

The division of society into "the rich" and "the poor" places my model into the field of models sometimes known as "class models", which originates in the work of Pasinetti (1962) and was adapted in papers such as Stiglitz (1967), amongst others. I will often make the simplifying assumption that the rich leave bequests whilst the poor consume over their lifetimes, and in this I am taking up an idea introduced by Baranzini (1991). The OLG model with which I finish this thesis then makes the extension that the poor are two-generation OLG agents, whereas the rich are infinitely lived, and this is an idea borrowed from Michl (2007).

An example of a recent usage of this kind of model is Mattauch et al. (2016), where such a model is used to show how capital taxation can be welfare improving in a world where public capital is provided.

2.2 Asset Prices

Recent rapid increases in asset prices have been covered in a variety of works by a number of economists. Most notably, is the recent work Knoll et al. (2017) who analyses house prices empirically across a long time period for fourteen advances economies.

2.3 Inequality

Thomas Piketty has done much work on recent increases of wealth inequality and its potential causes. Most well known is, of course, his book "Capital in the 21st Century" (Piketty, 2018), in which Piketty provides much data about the recent increases in wealth inequality, and some analysis of the possible causes. This book provides a story of increasing wealth inequality in developed nations over the last thirty years, a case supported by the World Inequality Report 2018 (Alvaredo et al., 2018). There was some critical response to Piketty's work that much of the increases in wealth inequality which he recorded were caused largely by increases in house prices. The most notable example of this is the criticism of Rognlie (2014). The work of this thesis interacts with both this work and its criticism by demonstrating a mechanism by which wealth inequality could cause an increase in asset prices, which, in turn, could be viewed as causing an increase in wealth inequality.

2.4 Savings behaviours of the rich

This thesis will utilise utility functions that deliver a marginal propensity to consume that is diminishing in wealth. This technique is supported by a recent paper by Straub (2018) which empirically supports the idea that the rich have a lower marginal propensity to spend than the poor. This builds on previous work by Attanasio (1994) and Dynan et al. (2004) which both also provide empirical support for the idea that the rich have a lower marginal propensity to consume. The assumptions that I will sometimes make about differing dynastic savings behaviour of the rich and poor (that the rich save dynastically whilst the poor are life-cycle savers) are supported empirically by data in Saez and Zucman (2016). Such an argument can obtain strong support from Federal reserve statistics – the 2016 Federal Reserve Survey of Consumer Finances indicates that the median left inheritance (of those who left an inheritance) in the USA was \$69,000, whereas the mean average was \$707,291. The mean average of those who left a trust fund was \$4,062,918 (US Federal Reserve, 2016).

The appearance of wealth directly in the utility function has much precedent, most recently in Michaillat and Saez (2018), which also includes a good account of the history and justifications of such a utility function. Notably they cite Irving Fisher, to whom the modern view of saving as a way to optimize one's consumption stream is often attributed, as providing support for the idea in Fisher (1930).

2.5 The effect of wealth inequality on asset prices and interest rates

There appears to be very little work on the potential relationship between wealth inequality, asset prices and interest rates. I have only been able to find one paper by Grüner (2001), an empirical study of the relationship between wealth inequality and interest rates. The paper finds a weak positive relationship between wealth inequality and interest rates, which conflicts with the general results of this paper, when considering 18 data points between 1911 and 1983. Given the wide time range of the data set, and the large number of variables which could have affected both wealth inequality and interest rates over the period of time, I do not believe that this empirical study has persuasively proven any definite relationship between the two variables.

2.6 Other papers assessing the macroeconomic impact of wealth inequality

Whilst there has been very little work connecting wealth inequality with asset prices and interest rates, there has been significant work analysisng the potential for a connection between inequality more broadly and growth. Results of empirical papers have been conflicting, with the work of Ostrey and Berg (2014), asserting that reducing inequality can be positive for growth, whereas Forbes (2000) asserts that income inequality demonstrates a positive relationship on later growth. There are also theory papers which attempt to tackle the question, with the work of Corneo (2001) notable both for employing some measure of wealth directly in utility functions, similarly to this thesis, and for then concluding that greater wealth inequality leads to lower savings, directly in contrast to this paper.

In general, whilst there has been much work on inequality, there has been virtually nothing analysing the specific relationship that wealth inequality can have on asset prices or interest rates. I hope this thesis can go at least some way towards filling that gap.

Chapter 3

Assumptions and concepts

3.1 Asset accumulation equations

My model will explore the possibility that wealth inequality has an effect upon asset prices and interest rates due to a marginal propensity to consume which is decreasing in wealth. Throughout, my models will be "real" models, in the sense that I will not at any point include money in the model. I will normalise the price of the consumption good, and thus the concept of "asset price" which will be explored will be the relative price of assets compared to the price of the representative consumption good.

Since my interest is in the relative price of assets and consumption, I will not be able to use traditional capital accumulations of the form:

$$K_{t+1} = K_t + Y_t - C_t \tag{3.1}$$

Where Y is output, C is consumption, and K is capital which can be considered the "asset" in this case, and t subscripts indicate time.

Equations of this form imply that the consumption good and the capital good can be freely transformed into one another. When a model allows for this free, bidirectional transformation, there can be no space for interesting movements in the relative prices of the two goods. Equations of this sort are not suitable for models interested in changes in this relative price.

In reality, some assets are reproducible and some assets are not reproducible and, in fact, a great many number of assets, such as a townhouse in a prime London location, are some mixture of the two. The simplest way in which we can explore the relative prices of consumption and asset goods, is to assume that asset goods are fixed and non reproducible. I will start with this assumption, and will later loosen this to allow for two types of assets - reproducible and non-repoducible, in which case I will be interested in the price of the non-reproducible asset. In order that it is always clear exactly which kind of asset is being discussed, I will henceforth use K (capital) for reproducible assets, T (as in "terra" or "land") for non reproducible assets in models where both reproducible and non reproducible assets exist, and W in simple models with only one, non reproducible asset, to represent all forms of "wealth".

This assumption can be interpreted in a variety of ways. It can be considered to be referencing only the non-reproducible portion of capital, or it can be considered to be with reference to a shorter time frame, in which there is not enough time for new capital to be reproduced. It could also be considered to be referencing a mature economy which has reached ecological bounds and within which non-reproducible, rather than reproducible, assets have started to be the limiting factor on the economy. Most pertinently, I believe that fixed capital models could be very relevant in economies, such as those in most of the world at present, where interest rates have been stuck at the zero lower bound for long periods of time, implying that saving exceeds investment and excess savings are unable to be converted into capital. This is an essential assumption for exploring this question.

Eliminating the free transformability between the consumption good and the capital good means that it becomes very important to be clear at all times which quantities are in terms of the consumption good and which are in terms of the capital good.

3.2 Diminishing marginal propensity to consume

General Equilibrium models most commonly feature utility functions of the form:

$$U = \sum_{t=0}^{\infty} \delta^t f(C_t) \tag{3.2}$$

(Where U is utility, δ is a discount factor and f is an increasing function of consumption)

Utility functions such as these do not typically have the feature that the marginal propensity to consume an extra unit of income falls as the income of an individual increases, if that increase in income is permanent. I believe that this is an unrealistic feature of this utility function, and that, in reality, people with higher wealth levels have, in general, lower propensity to consume incremental income. This is also supported by empirical evidence from Straub (2018).

The mechanism which I am seeking to explore in this thesis, is that the diminished marginal propensity to consume of very rich people causes demand for assets to increase when wealth inequality increases. As such, I will have to use a different utility function. In general I will make the assumption that the utility of the wealthy depends explicitly on both consumption and wealth saved after the consumption decision is made, broadly taking the form $U(C, W_s)$ where W_s is a measure of post-consumption decision wealth. The function will be assumed to have positive first and negative second derivatives in both arguments, and diminishing marginal propensity to consume in wealth can be achieved by using a function such as:

$$U(C, W_s) = \ln C + \sqrt{W_s} \tag{3.3}$$

Which I will frequently use throughout the thesis. Using this function gives a marginal propensity to spend income on consumption (as opposed to saving) decreasing continuously from 1 when total income available to spend on both is 0, to 0 as total income available to spend approaches infinity, regardless of the price of the two assets.

These are the two key assumptions that I have made in order to explore these phenomena. All other assumptions are in line with standard general equilibium models. With that, I will begin by introducing the simplest version of the model.

3.3 Interest rates

The "interest rate" on an asset, is generally considered, within simple economic models, to be the per-period return, in units of consumption goods awarded to each unit of the asset, which is most commonly capital. When considering the meaningfulness of the interest rate, it must be clarified whether there is a mismatch of units between the asset and its return.

Interest rates are often considered to be percentages, yet this is not technically correct if we have a mismatch of units - if one house yields a return of £7,000 in one year, it is not correct to say that the house has an annual yield of 7,000%. In order to express two relative quantities in terms of a percentage, it is important that the two quantities are expressed in the same units first. Thus, to calculate the yield on a house, it is most sensible to multiply the number of houses first by the price of a house (in a certain currency), before comparing it to the rent of the house (in the same currency).

Since many economic models treat capital and consumption goods as interchangeable, this comparison of quantities of differing units, is often not problematic. But in this paper, where I am primarily interested in changes in the relative prices of capital and consumption goods, it will be important to express interest rates clearly correctly.

Firstly, it should be noted that the term frequently referred to as the interest rate, and denoted as r, which results from the derivative of the production function with respect to an asset, is not strictly a percentage. It it is a return, in consumption goods, on a unit of the asset. Throughout this paper, I will use the term r to refer to this quantity, but it will never be a percentage - it will be the price, in consumption goods, paid to rent one unit of the asset.

In order to express the return on an asset in terms of a percentage, we need simply to divide the term r by the price of the asset, which I shall always refer to as p. I will refer to this percentage return $\frac{r}{p}$ as i throughout. You may consider r to be the *real* return on a unit of an asset, and i to be the *percentage* or *effective* return on the asset. Note that if we are to consider any measurement to be an "interest rate" insofar as an interest rate is a percentage, we must consider i. All interest rates in the real world, be they interest rates on bonds, rental yields on houses, or dividend yields on bonds, are values of i rather than r.

3.4 The Inequality Mechanism

As my interest is primarily with the effects of inequality, I will clearly be unable to use a pure representative agent model. As such, I developed a very simple mechanism to represent and vary inequality with only two agents, which is to divide society into a fraction of size E which is "rich" and a fraction $1 - E$ which is "poor". All assets are then divided equally between the "rich" group, whilst the poor are given no assets. Both rich and poor agents are assumed to work and, for simplicity, to each provide a constant 1 unit of labour per person. The variable E is chosen to represent "Equality" and can exist within the range (0,1], where $E = 1$ would represent classic "everybody is equal" representative agent and E approaching 0 would represent ever smaller portions of the population owning all wealth. E cannot be 0 as there must exist a non-zero mass of rich people to own the wealth. This is clearly a gross simplification of a wealth distribution, but it allows us to vary inequality within a simple model, and represents the situation of a vast majority of wealth being held by a small minority of a society, which appears to be common in many countries, and, perhaps, the world (Benhabib and Bisin, 2018) (Wolff, 2017).

For mathematical simplicity, I will assume the total size of society is 1 throughout. This means that there will always be less than 1 rich agent and less than 1 poor agent, and, as such, when we look at "individual" quantities for either the rich or poor agent, they will be greater than "aggregate" quantities for the rich or poor agents. I am pointing this out now to avoid any confusion. In reality, E can be thought of as the percentage of society who owns all or the vast majority of the society's wealth.

This covers all of the new concepts in the model which require introduction, so I will now move on to introducing the most basic form of the model.

Chapter 4

Static Model

In this first, most basic form of the model, there is only one asset W, for Wealth. Aggregate wealth is $\bar{W}$. I shall use bar notation to refer to aggregates throughout this thesis. The wealth split evenly between the rich and thus the individual rich person has an amount of wealth $\frac{\bar{W}}{E}$ which I shall call W_i representing the "inherited" wealth of each individual rich person. These are all in units of wealth.

Rich and poor provide L_r and L_p units of labour respectively. For simplicity I currently assume both are L. This assumption will be continued throughout this thesis. These are all in units of labour.

Poor consume all income. This is clearly a gross simplification of the behaviour of poor people, but is maintained for mathematical simplicity at this point. It will be relaxed later in the thesis. Rich choose between consumption and increasing their "saved wealth" W_{sr}, where W_{sr} is defined as the individual amount of wealth that they have after making their consumption decision.

Timing is as follows: The rich receive their inherited wealth, their labour income and their wealth income. Labour income and wealth income are both determined by the normal supply side equilibrium conditions, which I will explain later, and are paid in units of the consumption good. They then enter into the market for wealth and the consumption good. Relative price adjusts in a Walrasian fashion to clear both markets. I will normalise the price of the consumption good and use p for the price of the wealth good. The price p will thus be in units of the consumption good.

We thus have the following definition for the saved wealth of the individual rich person, W_{sr}, which can also be considered "the budget constraint":

$$W_{sr} = \Big(1 + \frac{r}{p}\Big)W_i + \frac{w}{p}L - \frac{C_r}{p} \tag{4.1}$$

Where:
$W_i = \frac{\bar{W}}{E}$
$L =$ Labour supply
$C_r =$ Per person consumption of the rich
$r =$ return on one unit of wealth
$w =$ return on one unit of labour
$p =$ cost of one unit of wealth

W_{sr} and W_i are measured in units of wealth. r, w, C_r and p are all measured in units of the consumption good. L is measured in units of labour. Since the total number of people is 1, the amount of work done by the individual rich, the amount of work done by the individual poor and the total amount of work done are all exactly L.

I then specify both the production function, and the Utility function of the rich, both of which will be generalised later. The specific functions I chose were as follows:

$$U_r = \ln C_r + W_{sr}^{\frac{1}{2}} \tag{4.2}$$

and

$$Y = A\bar{W}^a L^{1-a} \tag{4.3}$$

Where U_r, Y, A and a are utility of the individual rich, output (in terms of the consumption good), a technology parameter and the labour share of income, respectively, completely as a standard Cobb-Douglas production function. A is positive and a is in [0,1].

Market clearing in the consumption good, recalling that a mass of (1-E) poor people consume all their labour income:

$$Y = EC_r + (1-E)wL \tag{4.4}$$

Market clearing in wealth is simply:

$$EW_{sr} = \bar{W} \tag{4.5}$$

We also have the standard market clearing conditions in the rental markets for labour and wealth:

$$w = A(1-a)\Big(\frac{\bar{W}}{L}\Big)^a \tag{4.6}$$

$$r = Aa\Big(\frac{L}{\bar{W}}\Big)^{1-a} \tag{4.7}$$

This specifies the model completely, which can now be solved to investigate the relationship between p or $\frac{p}{w}$ and E.

4.1 Solving the basic static model

The solution of the spending-saving problem of the rich individual can be found by substituting (4.1) into (4.2) and maximising as a function of C_r. This gives the following expression for the chosen consumption of the individual rich:

$$C_r = 2p[(1 + (1 + \frac{r}{p})W_i + \frac{w}{p}L)^{\frac{1}{2}} - 1] \tag{4.8}$$

Substituting the market clearing level for wealth from (4.5) into the definition of W_{sr} which is equation (4.1), we can derive the following expression for the equilibrium level of C_r:

$$C_r = W_i r + Lw \tag{4.9}$$

Substituting (4.9) into (4.8) to eliminate C_r and rearranging to make p the subject gives:

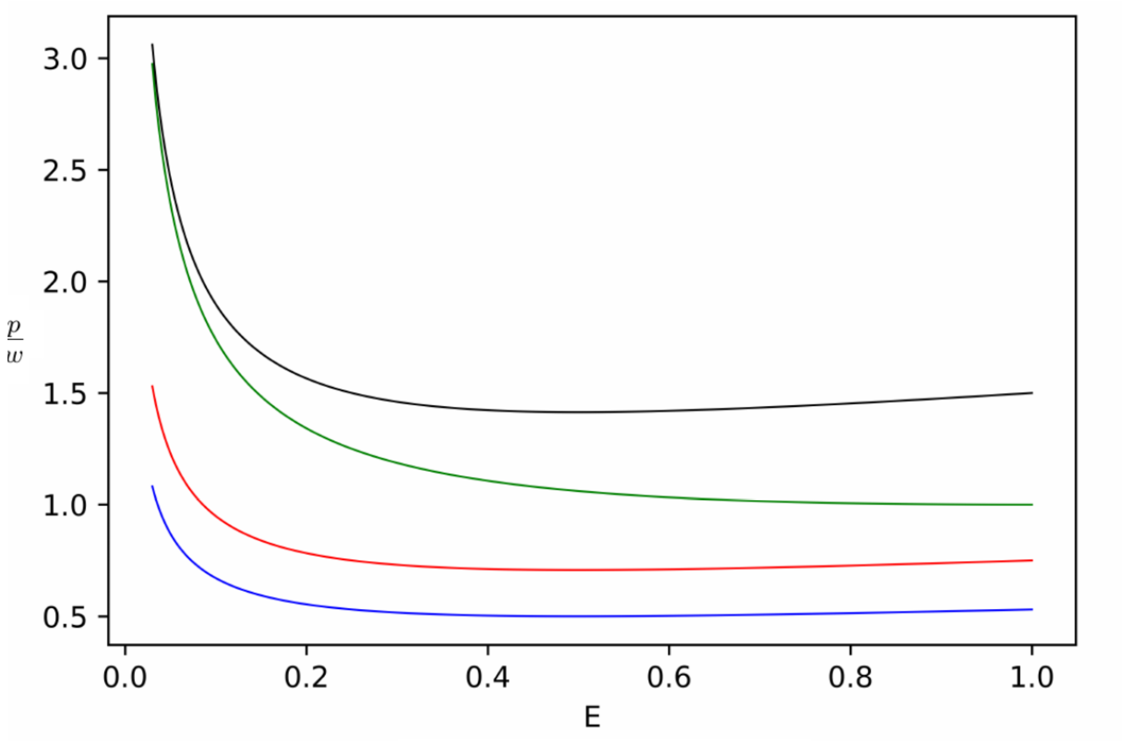
$$p = \frac{wL}{2}\Big(\frac{E}{\bar{W}}\Big)^{\frac{1}{2}} + \frac{r}{2}\Big(\frac{\bar{W}}{E}\Big)^{\frac{1}{2}} \tag{4.10}$$

Now, by substituting out w and r using equations (4.6) and (4.7), we can obtain an expression for $\frac{p}{w}$ in terms of only the exogenous parameters of the model. $\frac{p}{w}$ is the asset price as a multiple of the wage, and thus gives us a sense of the "affordability" of assets. The expression is as follows:

$$\frac{p}{w} = \frac{L}{2\sqrt{\bar{W}}}\Big(\sqrt{E} + \frac{a}{1-a}\frac{1}{\sqrt{E}}\Big) \tag{4.11}$$

The relationship between $\frac{p}{w}$ and E is show graphically in 4.1 below for a variety of parameter combinations.

Note that $\frac{p}{w}$ shows interesting dynamics as a function of E on the range of $E \in (0, 1]$. $\frac{p}{w}$ tends to infinity as E tends to 0, regardless of parameter values chosen, with the single exception of $a = 0$. This can also clearly be observed from equation (4.11). As E increases $\frac{p}{w}$ decreases sharply, before reaching a minimum, and then starting slowly to increase. Changes in W and L do not have interesting



Black: $a = \frac{1}{3}, \bar{W} = 1, L = 2$
Green: $a = \frac{1}{2}, \bar{W} = 1, L = 1$
Red: $a = \frac{1}{3}, \bar{W} = 1, L = 1$
Blue: $a = \frac{1}{3}, \bar{W} = 2, L = 1$

Figure 4.1: Graph of $\frac{p}{w}$ against E

effects, simply scaling the price of wealth down and up respectively as they rise. The capital share of income, a, however, has a key effect upon the shape of the graph. The value of E which gives the minimum $\frac{p}{w}$ is a function of a. $a = 0.5$ sees a minimum at $E = 1$, thus $\frac{p}{w}$ is decreasing in E across the whole range of E. $a = \frac{1}{3}$, however, sees a minumum at $E = 0.5$, and the range of E for which $\frac{p}{w}$ is agressively decreasing is smaller.

Data from the OECD (2012) suggests the labor share of income was dropping from 66.1% to 61.7% from 1990 to 2009, suggesting an α of about 0.38. Data from Benhabib and Bisin (2018) and Wolff (2017), both suggest wealth distributions to be heavily concentrated amongst the richest in society, indicating that the most relevant portion of the graph would be for low levels of E.

Differentiation of $\frac{p}{w}$ with respect to E and with respect to a can describe more precisely how it is affected by those parameters:

$$\frac{d(\frac{p}{w})}{dE} = \frac{L}{4\sqrt{\bar{W}}}\left(E^{-\frac{1}{2}} - \frac{a}{1-a}E^{-\frac{3}{2}}\right) \tag{4.12}$$

Which is positive if and only if $E > \frac{a}{1-a}$ and negative otherwise. $E = \frac{a}{1-a}$ will thus be the point at which increasing equality starts to cause asset prices to rise. We can see there that if $a = 0$ the derivative is positive for all positive values of E.

$$\frac{d(\frac{p}{w})}{da} = \frac{L}{2\sqrt{\bar{W}E}(1-a)^2} \tag{4.13}$$

Which is always positive for the acceptable range of a between 0 and 1 and is decreasing in E. Thus increasing capital share of income can, understandably, be expected to cause asset affordability to decrease, but this effect can be ameliorated by less inequality.

Whilst it is clear from 4.1 that $\frac{p}{w}$ is decreasing in E for low levels of E, and aggressively so for very low E, the nature of the relationship for levels of E closer to one is less strong and, directionally, a function of a. This can clearly be seen in fig 4.2 below, which analyses the behaviour of $\frac{p}{w}$ as a function of E for $a = \frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively, reading from the top:

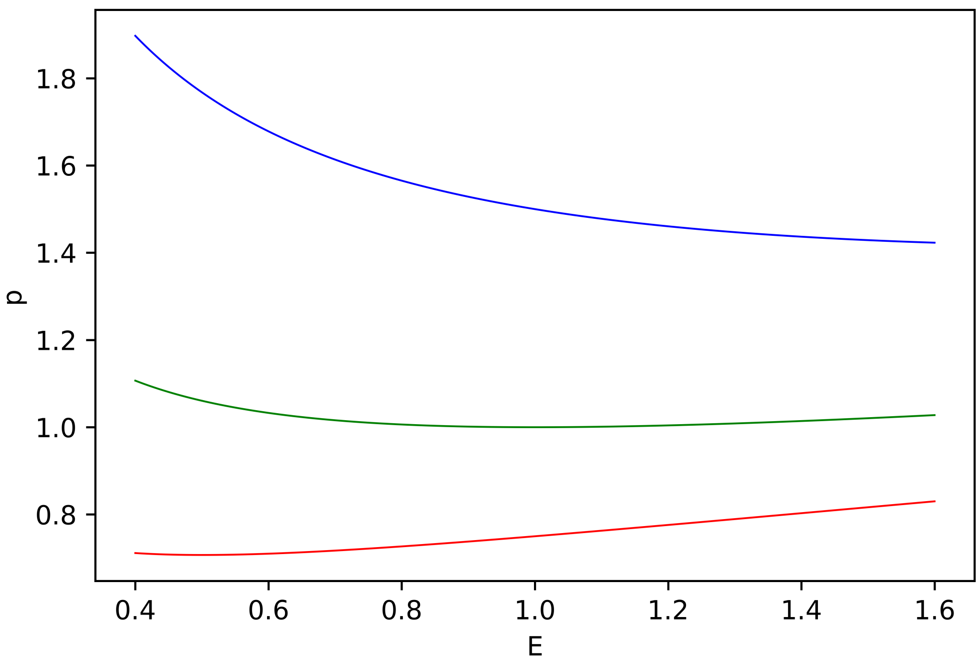


Figure 4.2: Graph of $\frac{p}{w}$ against E

This graph clearly shows that the relationship of $\frac{p}{w}$ is ambiguous at the top end of the feasible range of E, decreasing for high levels of a, increasing for low levels of a, and constant at $E = 1$ for exactly $a = \frac{1}{2}$.

The effective interest rate i in this model is equal to $\frac{r}{p}$, which is equal to the inverse of $\frac{p}{w}\frac{w}{r}$. Since r and w are both constant here, i is the inverse of $\frac{p}{w}$, multiplied by a constant, and thus demonstrate an inverse relationship to E to that of $\frac{p}{w}$. i tends to 0 as E approaches 0, and finds maxima exactly where $\frac{p}{w}$ finds minima, for a given a. The exact expression for the interest rate is as follows:

$$i = \frac{r}{p} = \frac{2}{\sqrt{\bar{W}}}\Big(\frac{1-a}{a}\sqrt{E} + \frac{1}{\sqrt{E}}\Big)^{-1} \tag{4.14}$$

4.2 Discussion of results

The model yields the following results. As wealth becomes concentrated in fewer hands, the individual rich become richer. Due to the choice of utility function, with richer individuals preferring wealth over consumption in comparison to poorer individuals, this increases the overall societal preference for wealth over consumption at any given relative price of the two goods. Since both the wealth good and the consumption good are in fixed supply in the economy, this means that the price of the wealth good has to rise as equality falls towards zero.

However, this is not the case for all values of the a (capital share of income) and E parameters. For $a = 0.5$, p is decreasing in E for all acceptable values of E, but for $a = \frac{1}{3}$, p is increasing in E, albeit slowly, for large values of E. This clearly shows that there are two conflicting forces in effect.

As E increases, the number of poor in society decreases, and the number of rich increases. In effect, increasing E "transforms" poor people into rich people. This decreases the level of inequality within society, but, due to the fact that the poor and the rich have different utility functions in this model, it also changes the behavioural preferences of the individuals who have transformed. The poor in this model consume all income. The rich, on the other hand, always choose a combination of consumption and savings. As such, when a poor person is transformed into a rich person, it inherently increases society's overall propensity to save, thus pushing p, the cost of the savings good relative to the consumption good, up.

This describes the two contradictory effects of an increase in equality. As equality increases, the rich become less rich and thus move to preferring consumption over savings. This pushes p down. As equality increases, poor people "flip" into rich people, and thus flip to more conservative savings behaviours, pushing p up. Since all capital income goes to the rich, higher levels of a weight the first effect more heavily than the second.

For high levels of E, the utility functions chosen are somewhat unrealistic. When inequality is very high, and the rich are much richer than the poor, it is realistic to assume that the rich save a much higher proportion of their income than the poor. However, if inequality is low, and each individual rich person owns only a moderate level of wealth, it is longer realistic to assume that the rich save a much greater proportion of their income than the poor do. Indeed, such an assumption may lead to the unrealistic conclusion that the rich have a lower absolute level of consumption than the poor, especially if the capital share of income, a, is low. For $a = 0$, for example, it can clearly be shown that the consumption of the rich is unambiguously less, in absolute terms, than that of the poor (since the poor consume all income, whereas the rich save some income, and incomes of both agents are the same). I suspect that the non monotonicity of p with respect to E may will be a result of this inaccuracy. I will thus, in the next chapter of this thesis, present an extension of this model where both agents share the same utility function. It will be seen in that case that p is uniformly decreasing in E.

High levels of wealth inequality lead to very low levels of i. This could perhaps be illustrating a reason for the repeatedly surprising persistence of low interest rates in recent years, which may be a consequence of growing levels of wealth inequality. When the rich are very rich, they attempt to invest a great portion of their income. If available investments are limited, such as in this model, this could push up the price of investments, pushing down interest rates. In a globalised economy, any central banks who attempt to buck the low interest rate trend may find themselves swamped with capital inflows, pushing their currency up until they are forced to lower interest rates again. Mathematically, the behaviour of i is simply the inverse of the behaviour of p, since $i = \frac{r}{p}$ and r is, here, fixed.

It can also be seen that, using these specific functions, the relationship between the variables of interest, p, $\frac{p}{w}$, and i with the technology parameter A is uninteresting. In fact, A has no effect on any of these variables at all. This is actually a quirk of the specific functions chosen in this model, which I will now demonstrate by presenting a generalised form of the model.

4.3 Generalisation of the static model

The model and its results can be further analysed through generalising the specific functional forms used.

Let us define a general additive utility function for the rich as a function of consumption and wealth savings:

$$U_r = f(C_r) + g(W_s) \tag{4.15}$$

The production function can also be generalised as follows:

$$Y = Y(A, \bar{W}, L) \tag{4.16}$$

From these, we can find generalised solutions for p, i and $\frac{p}{w}$.

Solution of the spending/savings decision of the individual rich gives the following optimality condition:

$$p = \frac{g'(W_s)}{f'(C_r)} \tag{4.17}$$

Into which can be substituted can be substituted the definition of W_{sr} (4.1) to give the following:

$$p = \frac{g'[(1 + \frac{r}{p})W_i + \frac{wL}{p} - \frac{C_r}{p}]}{f'(C_r)} \tag{4.18}$$

This can be equated with the previously derived equilibrium condition for C_r equation (4.9) to give:

$$p = \frac{g'(W_i)}{f'(wL + rW_i)} \tag{4.19}$$

If we introduce the following notation for marginal products of wealth and labour respectively:

$$Y_L = \frac{dY}{dL} \tag{4.20}$$

$$Y_W = \frac{dY}{d\bar{W}} \tag{4.21}$$

And substitute $\frac{\bar{W}}{E}$ for W_i, then we derive the following expression for p:

$$p = \frac{g'(\frac{\bar{W}}{E})}{f'(Y_L L + Y_W \frac{\bar{W}}{E})} \tag{4.22}$$

With $\frac{p}{w}$ being simply:

$$\frac{p}{w} = \frac{g'(\frac{\bar{W}}{E})}{Y_L f'(Y_L L + Y_W \frac{\bar{W}}{E})} \tag{4.23}$$

4.3.1 The effects of technological improvement

A close analysis of this equation reveals that the invariance of p and thus $\frac{p}{w}$ with A does not hold in the general case. The numerator is always independent of A, but the denominator will, in general vary with A. If the production function is of the form $Y(A, \bar{W}, L) = AZ(\bar{W}, L)$, such as in the case of the Cobb-Douglas production function used, then the denominator, and thus the whole expression, will only be independent of A in the case where the utility function is such that $f(x) = \ln x$, or has the same first derivative.

If we make the assumption that output is always increasing in both W and L, then it can clearly be seen that both p and w are always increasing in A. If we further limit ourselves to production functions of the above form, where A is a multiplicative factor and thus $Y(A, \bar{W}, L) = AZ(\bar{W}, L)$, whether $\frac{p}{w}$ will be decreasing or increasing in A will depend, essentially, on the curvature of the f function, with $f(x) = \ln x$ being the borderline case. Where $f(x)$ is *more* curved than $\ln x$, meaning that utility from consumption is relative rapidly satiated, $\frac{p}{w}$ will be increasing in A. Where $f(x)$ is *less* curved than $\ln x$, meaning that utility from consumption is relatively slowly satisfied, $\frac{p}{w}$ will decrease in A.

To express the above mathematically, if we consider lnC_r to be the specific limiting case of the following family of functions as s goes to 1:

$$U = \frac{1}{1-s} C_r^{1-s} - \frac{1}{1-s} \tag{4.24}$$

Then it can be seen that, given a production function linear in A, for any s greater than 1 (ie. utility of consumptions diminishes more quickly than lnC), $\frac{p}{w}$ is increasing in A, and for any s less than 1 (ie. utility of consumptions diminishes less quickly than $\ln C_r$), $\frac{p}{w}$ is decreasing in A. $\ln C_r$ is actually precisely the borderline case, for which $\frac{p}{w}$ is precisely invariant to A.

This is an interesting result, as it raised the possibility that technological improvements, even ones that explicitly increase wages, could potentially exacerbate problems of low asset affordability. Of course, if we were to allow for forms of technological change that were biased away from labour, this effect would be made even stronger.

4.3.2 The effects of changes in equality

Moving on to a deeper analysis of the relationship between asset affordability and equality in the general case, under the assumption that w and L are invarying with E, we can derive the following result:

$$\frac{d(p/w)}{dE} = \frac{\bar{W}[g'(\frac{\bar{W}}{E})Y_W f''(Y_L L + Y_W \frac{\bar{W}}{E}) - g''(\frac{\bar{W}}{E})f'(Y_L L + Y_W \frac{\bar{W}}{E})}{E^2 Y_L [f'(Y_L L + Y_W \frac{\bar{W}}{E})]^2} \tag{4.25}$$

This expression is, unfortunately, not particularly intuitive in and of itself. However, since $\bar{W}$ and the denominator are both always positive, it does allow us to express conditions for the sign of the expression which, as we have seen from the specific case, can be either positive or negative depending upon parameters.

The expression will be negative if and only if:

$$\frac{g''(\frac{\bar{W}}{E})}{g'(\frac{\bar{W}}{E})} > Y_W \frac{f''(Y_L L + Y_W \frac{\bar{W}}{E})}{f'(Y_L L + Y_W \frac{\bar{W}}{E})} \tag{4.26}$$

Care should be taken to note that, under normal assumptions that both f and g functions have always positive first and negative second derivatives, both fractions are negative. Thus it may be more intuitive to consider the inequality in the following form:

$$\left|\frac{g''(\frac{\bar{W}}{E})}{g'(\frac{\bar{W}}{E})}\right| < Y_W \left|\frac{f''(Y_L L + Y_W \frac{\bar{W}}{E})}{f'(Y_L L + Y_W \frac{\bar{W}}{E})}\right| \tag{4.27}$$

If this inequality holds, then we will have a negative relationship between equality and asset expensiveness: increased equality pushes $\frac{p}{w}$ down.

The points at which derivatives are being taken in this inequality, $\frac{\bar{W}}{E}$ and $(Y_L L + Y_W \frac{\bar{W}}{E})$ are the equilibrium values for the individual savings and spending of the rich, respectively. The expressions $\left|\frac{g''(\frac{\bar{W}}{E})}{g'(\frac{\bar{W}}{E})}\right|$ and $\left|\frac{f''(Y_L L + Y_W \frac{\bar{W}}{E})}{f'(Y_L L + Y_W \frac{\bar{W}}{E})}\right|$ are the absolute risk aversions of the g and f functions respectively, at their equilibrium points. Absolute risk aversion is essentially a measure of curvature, and it is thus clear that the relative curvatures of the utility curves in savings and consumption, at the equilibrium points, are crucial to determining the directional effect of a change in equality on the asset price/wage ratio. If the utility function in consumption is more curved, indicating relatively rapid satiation from consumption, then price/wage ratios will decrease with equality. If the utility function in wealth is more curved, indicating relatively rapid satiation from wealth, then price/wage ratios will increase with equality. If utility is considered to be linear in wealth (or consumption), then increased equality will always push asset price/wage ratios down (up). It is worth noting that, within the assumptions of this model there are no interesting interactions between equality and wages directly (when wages are expressed in terms of consumption goods), and thus these interactions are all occuring directly through affects of E upon p.

The inclusion of Y_W on the right hand side of the inequality reminds us that the power of an increase in equality to push asset prices down has power proportional to share of income which is being paid exclusively to wealthholders. If this is very low, then the asset inflationary effect of increasing equality will dominate - that of it "transforming" the preferences of agents towards savings as they are transformed from poor to rich agents. In this situation, increased equality above a point will push $\frac{p}{w}$ up.

These results clearly show that the relationship between E and $\frac{p}{w}$, whilst being negative for the levels of E that one might assume to be realistic, are not unambiguous. In particular, for low levels of Y_W and high levels of E, we can see that $\frac{p}{w}$ is actually increasing with E. As previously mentioned, I believe that this may be a result of the assumptions of the model being unrealistic in this scenario. I move on to address this is in my next section.

Chapter 5

Consistent Behaviour Extensions to the Static Model

5.1 Motivation

The basic static model has revealed an ambiguous relationship between $\frac{p}{w}$ and E which is primarily dependent upon the levels of E. and the capital share of income In particular, when E, the level of equality, is high, the capital share of income, a in the specific form of the model, is low, $\frac{p}{w}$ increases as equality increases.

I believe that this result is resulting from a weakness in the assumptions of the model. I have assumed that the rich both spend and save, whereas the poor consume all income. This is an accurate description of many economies, where, in general, the poor very rarely save any considerable amount beyond the scope of their own lifetime, while the rich usually do. However, in a situation where equality is high, meaning, in the context of this model, that the individual rich do not hold very large amounts of wealth, and the capital share of income is low, meaning that that wealth is not earning the rich high incomes, it begins to become non-sensible to assume that the rich would save significantly greater portions of their income than the poor do, largely because the rich would not have significantly higher incomes than the poor. Such assumptions could lead to nonsensical outcomes, such as the rich consuming, in absolute terms, less than the poor.

This undesirable artifact of the model can be avoided by giving the poor and the rich the same utility functions. This utility function can maintain the characteristic that wealthier individuals have a greater preference for savings – such as in the $U_r = \ln C_r + W_{sr}^{\frac{1}{2}}$ function used in chapter 2, but, in being applied to both the wealthy and the poor individuals, will represent both poor and rich individuals choosing their spending/saving optimum directly as a result of their income. This means that, whilst “rich” individuals will retain a greater marginal preference for saving over spending, when compared to “poor” individuals, this will not be a fact simply imposed upon them by their type difference – behaviour will be completely consistent between types (they will have the same utility function), but differences will arrive simply from the differences in income generated by the differing initial allocations of wealth. I believe this to be a more realistic representation of the spending/saving behaviour of the poor, who, in general, fail to accumulate wealth not because of they do not desire it, but rather because the lion’s share of their incomes is taken up by necessary or very important consumption. This model also has the appealling feature that the behaviour of the two types converges as their situation converges. This is, particularly, much more realistic for low levels of the capital share of income.

I hope that, by extending the model in this way, I can explore where the non-monotonic relationship between $\frac{p}{w}$ and E is valid generally, or whether it is simply being caused by a bad simplifying assumption in the simplest from of the model.

5.2 The model

This form of the model is identical to the specific form of the static model in every way with the single exception that the poor will now have a utility function identical to that of the rich. Thus where we previously had only the following utility function:

$$U_r = \ln C_r + W_{sr}^{\frac{1}{2}} \tag{5.1}$$

We now also have the following utility function:

$$U_p = \ln C_p + W_{sp}^{\frac{1}{2}} \tag{5.2}$$

Where W_{sr} is defined exactly as previously, and W_{sp}, the saved wealth of the individual poor, measured in wealth units, is defined as:

$$W_{sp} = \frac{w}{p}L - \frac{C_p}{p} \tag{5.3}$$

Where C_p is, obviously, the consumption of the individual poor, in consumption units.

Walrasian market clearing conditions are thus as follows.

Market clearing in the consumption good:

$$Y = EC_r + (1-E)C_p \tag{5.4}$$

And market clearing in wealth:

$$EW_{sr} + (1-E)W_{sp} = W \tag{5.5}$$

Walras's Law tells us that if either one of these equations is satisfied, then both will be, and conditions for r and w are unchanged.

5.3 Solving

The spending/savings decisions of both agents can be simply solved, by substituting the definitions of W_{sr} and W_{sp} into their utility functions, and then maximising each with respect to the single variable C_r or C_p. This gives the following optimality conditions for C_r and C_p:

$$C_r = 2p[(1 + (1 + \frac{r}{p})W_i + \frac{w}{p}L)^{\frac{1}{2}} - 1] \tag{5.6}$$

and

$$C_p = 2p[(1 + \frac{w}{p}L)^{\frac{1}{2}} - 1] \tag{5.7}$$

Where W_i remains $\frac{W}{E}$, as before.

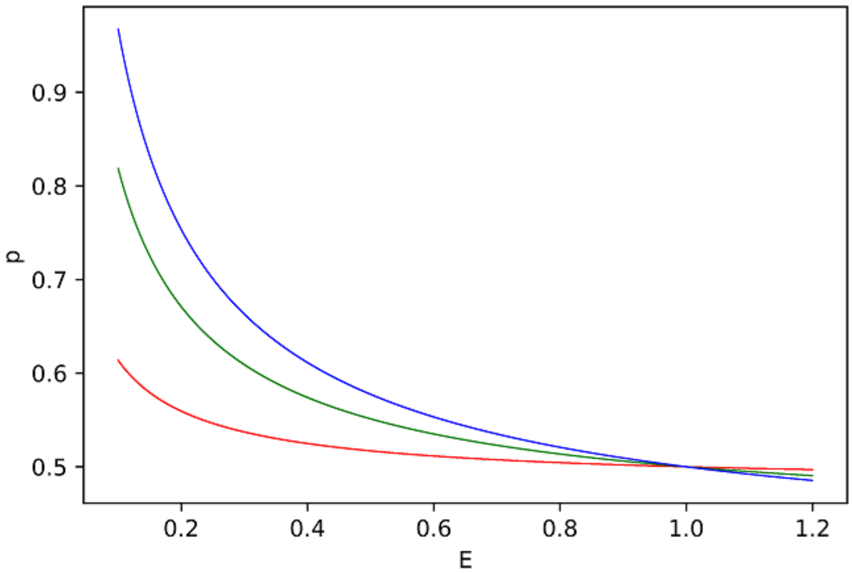
We can now solve for p by substituting these terms for C_r and C_p into the market clearing equation for consumption. This yields the following equation:

$$\frac{Y}{2p} = E[(1 + (1 + \frac{r}{p})W_i + \frac{w}{p}L)^{\frac{1}{2}} - 1] + (1-E)[(1 + \frac{w}{p}L)^{\frac{1}{2}} - 1] \tag{5.8}$$

Whilst I have not been able to rearrange this equation to make p the subject, it can be shown that it has a unique solution for p for each value of E (proof given in the appendix). If we substitute in our expressions for Y, r and w, we can derive the following equation consisting only of p and exogenous parameters:

$$\begin{aligned}&\frac{A\bar{W}^a L^{1-a}}{2p} - (1-E)\Big[(1 + \frac{A(1-a)(\frac{\bar{W}}{L})^a}{p}L)^{\frac{1}{2}}\Big] + 1 = \\ &E\Big[\Big(1 + \big(1 + \frac{Aa(\frac{\bar{W}}{L})^{a-1}}{p}\big)\frac{\bar{W}}{E} + \frac{A(1-a)(\frac{\bar{W}}{L})^a}{p}L\Big)^{\frac{1}{2}}\Big]\end{aligned} \tag{5.9}$$

This allows us to solve numerically for p as a function of E within the range of E, (0,1], (see appendix for a discussion of numerical solving). The results do not show interesting interactions between p and L, W or A. However, the relationship with a has fundamentally changed, as shown in 5.1 below.



Red: $a = 0$

Green: $a = \frac{1}{3}$

Blue: $a = \frac{1}{2}$

$W = L = A = 1$ in all lines

Figure 5.1: Graph of p against E

These graphs show us that, under the assumption that both agents share the same utility function, p becomes monotonically decreasing in E, regardless of the value of a. In fact, this is true even for $a = 0$, where wealth assets are non-productive and earn 0 rents. This validates my understanding that the previous dynamic of p increasing in E for low values of the capital share of labour and high values of equality, was caused by the differing utility functions of the two agents. If we believe that lower savings rates of the poor is a result of lower income of the poor, rather than any inherent differences, then it would make sense to believe that these are the better assumptions.

Not only does this extension of the model show that p can decrease monotonically with E, it also shows that, for all values of a, the values of p converge at exactly $E = 1$. .

The numerical solutions to the equation also show us that, within this extension to the model, the typical behaviour of p with respect to E, that of decreasing rapidly from infinity as E increases from 0, even holds when $a = 0$. This shows us that, if both poor and rich desire assets, inequality can push asset prices up even if those prices are completely unproductive. As it is not extremely clear in 5.1, I have included 5.2 below to show that, when $a = 0$ p continues to decrease as E extends up to and beyond $E = 1$.

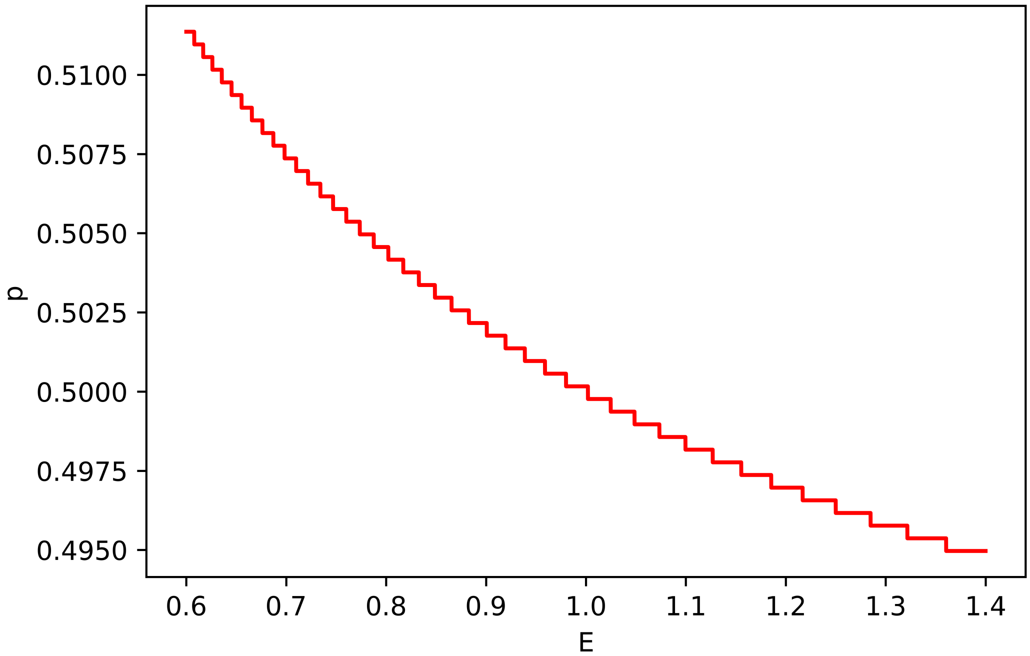


Figure 5.2: Close-up of $a = 0$ around $E = 1$

In the remaining iterations of my model, in order to simplify the algebra and solutions of the models, I will generally return to the simplifying assumption that the poor consume all income. This will often result in similar results to those given by the first presented iteration of the model. It will be important to recall the result given here, showing that non monotonicity in p in response to changes in E when E is large, can be caused by the discontinuous nature of the utility function between types.

I will now present, in the following chapter, an extended, infinite time horizon version of the model in which there is accumulable capital.

Chapter 6

The Dynamic Model

6.1 Introduction

6.1.1 Motivation

Having established the potential for relationships between wealth inequality, asset prices and interest rates through a static model, I now plan to extend the model into a dynamic model. This will allow me to introduce accumulable capital into the model, in a form which many economists will be familiar with. It will also allow me, later, to build an overlapping generations (OLG) extension to the model, to explore whether the high asset price, low interest rate scenario that could be caused by high wealth inequality could affect the ability of the poor to save for retirement.

6.1.2 Capital, Land, Wealth and their prices

As I discussed in chapter 2, accumulable capital, when accumulated through capital accumulation equations of the most common form, are not able to sustain interesting price variations in terms of the consumption good, since traditional capital accumulation equations tend to make these two goods, in some sense at least, bilaterally transformable into one another. To account for this, I will implement two forms of productive asset in the model; accumulable capital, which I shall call K throughout, and fixed land, which I shall call T, for "terra", throughout.

Since reproducible capital, K, and the consumption good, C are in some sense equivalent, as in most economic models, there will be no concept of a "price" of reproducible capital. I will employ a capital accumulation equation such that, in any time period t, C_t and K_t can be costlessly converted into one another, and thus the relative price of the consumption good and the capital good will always be 1.

The variable p_t will now refer only to the cost of the non reproducible asset, T_t called "land", and will be in terms of either the consumption good or the capital good, since both have the same price.

6.1.3 Utility functions

Now that we are dealing with an economy with more than one asset, we must discuss how to adapt the concept of rich individuals caring explicitly about "wealth". In the static model, with only one asset, W for "Wealth", which was fixed, I made utility explicitly a function of the fixed factor such that $U_r = f(C_r, W_{sr})$. The simplest extension would be to continue this logic exactly, allowing the rich agent to derive explicity utility from ownership of the fixed factor such that $U_{rt} = f(C_{rt}, T_{rt})$. Note here that I have added t subscripts to make clear that we are now in a dynamic, multi-period world.

Arguments could be made that wealthy people derive explicit utility from ownership of fixed assets, and hoarding of non-reproducible assets such as gold, land, luxury art and antique cars by the rich might support this idea, but many would argue that it is not explicitly non-reproducible assets that the rich desire, but rather some sense of "total net wealth", a concept which would better be represented by $K_t + p_t T_t$. Under this assumption, $U_{rt} = f(C_{rt}, K_{rt} + p_t T_{rt})$ would be a better utility function. I will consider both cases, starting with the $U_{rt} = f(C_{rt}, T_{rt})$ case.

6.2 The dynamic model with utility from land

For simplicity and ease of solving, I will now return to the simplifying assumption that the poor consume all income. I will thus drop the p and r subscripts on all variables - the individual poor simply consumes their entire income which is Lw_t in all periods, and all other variables are assumed to refer to the rich unless specified with a p subscript.

I consider the case where rich obtain utility explicitly from land specifically. Thus I will use the following utility function for the individual rich agent:

$$U = \sum_{t=0}^{\infty} \delta^t h(C_t, T_t) \tag{6.1}$$

Where δ is a constant, exogenous discount factor, and T_t is the amount of land owned by the individual rich agent in period t after making his spending/savings decision for that period. Note that, now that there are two assets, this decision is more complicated - the agent must choose not only how much to save, but how to allocate that savings between the capital asset and the land asset. This problem will be solved by introducing the variable B_t, which is defined as the amount of capital which is bought in period t in exchange for land. Thus B_t is in units of the capital good.

The introduction of this concept yields the following two "law-of-motion" equations for capital and land:

$$K_t = K_{t-1}(1 + r_t) + T_{t-1}\rho_t + w_t L - C_t + B_t \tag{6.2}$$

$$T_t = T_{t-1} - \frac{B_t}{p_t} \tag{6.3}$$

Where ρ_t is the return, paid in the consumption good, on one unit of land in period t.

These two equations describe completely the timing of the model, which I will describe briefly in words here, for clarity. Production takes place at the beginning of the period and rents are immediately paid on capital, land and labour based on the amounts of the factors which individuals held at the end of the previous period. After this, agents simultaneously choose both how much of their consumption good/capital (remember the two are the same) to consume and how much to save, and how much capital to sell/buy in exchange for land, which is the quantity known as B_t. Since total stock of land is fixed, the price p_t will adjust so that aggregate B_t is zero; since the poor consume all income, and thus do not participate in land or capital markets, B_t must be zero for the individual rich for the market to clear.

I set the Lagrangian up as follows:

$$\sum_{t=0}^{\infty} \delta^t \Big[h(C_t, T_t) + \lambda_t (K_t - K_{t-1}(1+r_t) - T_{t-1}\rho_t - w_t L + C_t - B_t) + \mu_t (T_t - T_{t-1} + \frac{B_t}{p_t}) \Big] \tag{6.4}$$

This yields the following two optimality conditions for the invidual rich agent, where $h_x(A, B)$ is the derivative of the h function with respect to variable x:

$$h_{C_t}(C_t, T_t) = \delta h_{C_{t+1}}(C_{t+1}, T_{t+1})(1 + r_{t+1}) \tag{6.5}$$

Which determines whether the agent should consume now versus consuming tomorrow.

$$h_{T_t}(C_t, T_t) + \delta\Big[h_{C_{t+1}}(C_{t+1}, T_{t+1})(\rho_{t+1} + p_{t+1})\Big] = p_t h_{C_t}(C_t, T_t) \tag{6.6}$$

Which determines whether and how much land the agent should buy/sell today.

6.2.1 Solving at steady state

At steady state all time subscripted variables are constant across time, thus time subscripts can be dropped from formulas (6.5) and (6.6) giving the following steady state equations:

$$r = \frac{1-\delta}{\delta} \tag{6.7}$$

$$p = \frac{1}{1-\delta}\Big[\frac{h_T(C,T)}{h_C(C,T)} + \delta\rho\Big] \tag{6.8}$$

Where the second equation is clearly the dynamic model equivalent of (4.17) equation from the generalised static model. For those familiar with the history of capital and land models, it will also be reminiscent of the classic result $r = \frac{\rho}{p}$ from the work of Feldstein (1977) and others. Here the classic result has been modified by the specific utility funcitons being used here, with the rich deriving explicit utility from land holdings.

6.2.2 General equilibrium

The solutions to the individual optimisation problem can be extended to find general equilibrium by imposing the following equilibrium conditions:

$$C = wL + \rho\frac{\bar{T}}{E} + r\frac{\bar{K}}{E} \tag{6.9}$$

$$B = 0 \tag{6.10}$$

$$T = \frac{\bar{T}}{E} \tag{6.11}$$

$$K = \frac{\bar{K}^*}{E} \tag{6.12}$$

Where I am again using bars such as $\bar{T}$ to indicate societal aggregates, and am introducing the $*$ subscript to indicate the steady state value for $\bar{K}$. $\bar{T}$ is fixed and exogenous, and $\bar{K}^*$ will be the steady state level of aggregate capital such that the marginal product of output with respect to capital (in an as yet unspecified production function) is equal to the value of r set by the exogenous δ as per the formula of equation (6.7).

This allows us to derive the following condition for price of land at equilibrium steady state:

$$p = \frac{1}{1-\delta}\Big[\frac{h_2[(wL+\rho\frac{\bar{T}}{E}+r\frac{\bar{K}^*}{E},\frac{\bar{T}}{E})]}{h_1[(wL+\rho\frac{\bar{T}}{E}+r\frac{\bar{K}^*}{E},\frac{\bar{T}}{E})]}+\delta\rho\Big] \tag{6.13}$$

Where h_x is defined to be the derivative of the function h with respect to the xth argument, for $x = 1, 2$.

At this, point, for simplicity, I again make the assumption that the utility function is additively separable and thus of the following form:

$$h(C_t, T_t) = f(C_t) + g(T_t) \tag{6.14}$$

Which allows the above to collapse to the much more manageable:

$$p = \frac{1}{1-\delta}\Big[\frac{g'(\frac{\bar{T}}{E})}{f'(wL+\rho\frac{\bar{T}}{E}+r\frac{\bar{K}^*}{E})}+\delta\rho\Big] \tag{6.15}$$

Which is clearly directly comparable to the following result from the generalised static model:

$$p = \frac{g'(\frac{\bar{W}}{E})}{f'(wL+r\frac{\bar{W}}{E})} \tag{6.16}$$

A quick comparison of these two terms for p shows that the dynamics driving price are similar in the static and dynamic models; within the dynamic model price will be generically higher, since land provides utility not just in the contemporaneous period, but in future periods too. The return on land plays an additional role, since it will now explicitly be received in the following period. Other than that, the relationship between p and E does not seem fundamentally changed, other than the amplification created by the extra savings motive.

6.2.3 The production function

The most general production function is:

$$Y_t = Y(A_t, \bar{K}^*_{t-1}, \bar{T}, L) \tag{6.17}$$

Where Y is a function of per period technology, per period capital (decided in the previous period), and total land and labour, which are both constant and non-varying with time.

These substitute into equation (6.16) in the obvious way:

$$p = \frac{1}{1-\delta}\Big[\frac{g'(\frac{\bar{T}}{E})}{f'(Y_L L + Y_T \frac{\bar{T}}{E} + Y_K \frac{\bar{K}^*}{E})} + \delta Y_T\Big] \tag{6.18}$$

Where Y_x is the derivative of the production function with respect to x at steady state equilibrium.

Having reached this equation, we can now begin to analyse the relationship of p to E in this model.

6.2.4 The relationship of p to E

Since L and $\bar{T}$ are fixed, if we assume that A_t is independent of E, then E cannot appear in the production function unless it affects the level of $\bar{K}^*$. $\bar{K}^*$ is the level of K such that the marginal rate of capital is r as specified in equation (6.7). It is therefore itself a function of $A, L, \bar{T}$ and δ, all of which are exogenous or assumed independent from E, and it is thus itself independent of E. This enables us to differentiate equation (6.18) for p with respect to E safe in the knowledge that E is only affecting p in the instances where it appears explicitly in (6.18).

We can thus complete the differentiation, which yields the following result:

$$\frac{dp}{dE} = \frac{g'\Big(\frac{\bar{T}}{E}\Big)f''\Big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\Big)\Big(Y_T\bar{T} + Y_K\bar{K}^*\Big) - \bar{T}g''\Big(\frac{\bar{T}}{E}\Big)f'\Big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\Big)}{E^2\big(1-\delta\big)\Big[f'\big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\big)\Big]^2} \tag{6.19}$$

For the reasons discussed above, the wage is not a function of E in this model. Thus it is trivial to extend this to $\frac{d(p/w)}{dE}$ as follows:

$$\frac{d\frac{p}{w}}{dE} = \frac{g'\Big(\frac{\bar{T}}{E}\Big)f''\Big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\Big)\Big(Y_T\bar{T} + Y_K\bar{K}^*\Big) - \bar{T}g''\Big(\frac{\bar{T}}{E}\Big)f'\Big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\Big)}{Y_L E^2\big(1-\delta\big)\Big[f'\big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\big)\Big]^2} \tag{6.20}$$

Which allows us to contrast directly with the following result from the single factor, static model:

$$\frac{d(p/w)}{dE} = \frac{W[g'(\frac{W}{E})Y_W f''(Y_L L + Y_W\frac{W}{E}) - g''(\frac{W}{E})f'(Y_L L + Y_W\frac{W}{E})}{Y_L E^2[f'(Y_L L + Y_W\frac{W}{E})]^2} \tag{6.21}$$

Most of the differences between (6.20) and (6.21) here are obvious; the equilibrium levels of consumptions and savings vary from $Y_L L + Y_W\frac{W}{E}$ to $Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}$ and from $\frac{W}{E}$ to $\frac{\bar{T}}{E}$ respectively between the two models, and thus these terms replace one another in the first and second derivative terms, as would be expected. The addition of $(1-\delta)$ in the denominator serves to unambiguously increase the power of E to change price due simply to the introduced significance of land in more than just the contemporaneous period and is not particularly interesting. There are, however, some slight differences to be noted when trying to ascertain the overall sign of the derivative.

As in the static case, both product terms in the numerator can be expected to be negative, as we can expect both f and g functions to have positive first, and negative second derivatives. Thus the overall derivative will be negative, implying increased equality pushes asset prices down, if the following inequality holds:

$$\Bigg|\frac{g''\big(\frac{\bar{T}}{E}\big)}{g'\big(\frac{\bar{T}}{E}\big)}\Bigg| < \big(Y_T + Y_K\frac{\bar{K}^*}{\bar{T}}\big)\Bigg|\frac{f''\big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\big)}{f'\big(Y_L L + Y_T\frac{\bar{T}}{E} + Y_K\frac{\bar{K}^*}{E}\big)}\Bigg| \tag{6.22}$$

Which can be directly compared to the following equation from the static model:

$$\Bigg|\frac{g''(\frac{\bar{W}}{E})}{g'(\frac{\bar{W}}{E})}\Bigg| < Y_W\Bigg|\frac{f''(Y_L L + Y_W\frac{\bar{W}}{E})}{f'(Y_L L + Y_W\frac{\bar{W}}{E})}\Bigg| \tag{6.23}$$

6.2.5 Analysis

The two inequalities are clearly very similar. The equilibrium points at which derivatives are taken are changed in the obvious way to fit the new assumptions of the model. The importance of the relative curvature of the two curves at equilibrium points remains, as before, but is now complicated by the existence of a slightly more complicated multiplicative factor. Additionally, in the basic, static, single factor model, Y_L, Y_W, W and L were all constant, whereas in the dynamic model, Y_L, Y_T and Y_K will all depend on the steady state value of $\bar{K}^*$ which will itself vary with δ and the specific production function chosen.

Let us consider then the differences between the two inequalities. Conceptually, W in the single asset model represents all assets, and thus encompasses what are both T and K in the two asset model. As such the points at which the f function is being differentiated, total income of the individual rich in both cases, are conceptually identical, and thus there is no meaningful difference between the right hand sides of both equations.

When we look at the left hand side, there are two differences to note. Firstly, since $\bar{W}$ represents all assets, whereas $\bar{T}$ represents only one subset of all assets, it is to be expected $\bar{T}$ represents a smaller quantity than $\bar{W}$. If we assume that the f function is unchanged in both cases, and that it has the typical features of a utility function, those being that $f'(x)$ is always positive but decreasing as x increases and that $f''(x)$ is always negative but decreasing in magnitude as x increases, it will be true that the g function will be more curved at $\frac{\bar{T}}{E}$ than it will be at $\frac{\bar{W}}{E}$. This represents the fact that "land", or non reproducible assets specifically, are scarcer than all assets generally. This effect could be expected to disappear if, for example, the rich received utility from relative land/wealth holdings rather than absolute holdings.

Secondly, we have the more complicated multiplicative factor $\left(Y_T + Y_K \frac{\bar{K}^*}{\bar{T}}\right)$ in the two asset model, as opposed to simply Y_W in the single asset case. Since W represents the sum of T and K, it can be expected that Y_W is equivalent to $Y_T + Y_K$, and thus that the expression $\left(Y_T + Y_K \frac{\bar{K}^*}{\bar{T}}\right)$ is equivalent to T_W in the case where $\bar{K}^* = \bar{T}$. If $\bar{K}^*$ is greater than $\bar{T}$, then increases in equality will be more likely to bring the price of land down, and if $\bar{T}$ is greater than $\bar{K}^*$, then increases in equality will be less likely to bring the price of land down. The importance of this factor is scaled by the income paid to capital - if capital is plentiful, and receives high income, and land is scarce, then an increase in equality decreases already low land holdings of rich individuals who have large incomes, due to the high rents paid to capital, leading to greater competition for land and increased land prices.

Whilst these differences will clearly have an effect on the relationship between p and E, especially when $\bar{K}^*$ and $\bar{T}$ are very unequal, the overall nature of the relationship is likely to be similar to that of the static case, and we can test this by graphing the relationship for the same specific functions which we used in the static case.

6.2.6 Graphing using specific functions

Clearly the previously used production function of $Y = A\bar{W}^a L^{1-a}$ is now invalid as we must add T as a third factor of production. I believe the natural extension to be:

$$Y = A\bar{K}^a\bar{T}^b L^{1-a-b} \tag{6.24}$$

Since W represented all assets in the single asset model, what was previously α will be equivalent to what is now $a+b$, so that the total share of income going to assets is approximately the same. For the f and g functions I will use the same functions as before which are $f(x) = \ln x$ and $g(x) = \sqrt{x}$.

These equations yield the following equation for p:

$$p = \frac{1}{2(1-\delta)}\Big[wL\big(\frac{E}{\bar{T}}\big)^{\frac{1}{2}} + (\rho\bar{T} + r\bar{K}^*)\big(\frac{\bar{T}}{E}\big)^{\frac{1}{2}} + 2\delta\rho\Big] \tag{6.25}$$

By taking marginal products of the production function we have the following three terms for w, r and ρ:

$$w = (1-a-b)A\bar{K}^a\bar{T}^b L^{-a-b} \tag{6.26}$$

$$r = aA\bar{K}^{a-1}\bar{T}^b L^{1-a-b} \tag{6.27}$$

$$\rho = bA\bar{K}^a\bar{T}^{b-1}L^{1-a-b} \tag{6.28}$$

Equation (6.27) can be combined with equation (6.7) to provide the following term for $\bar{K}^*$:

$$\bar{K}^* = \Big(\frac{1-\delta}{\delta aA\bar{T}^b L^{1-a-b}}\Big)^{\frac{1}{a-1}} \tag{6.29}$$

These can all be substituted into equation (6.25) to give the following expression for p in terms of only E and exogenous parameters:

$$p = \frac{1}{2(1-\delta)}\Big[\frac{\delta aA\bar{T}^b L^{1-a-b}}{1-\delta}\Big]^{\frac{1}{1-a}}\Big[(1-a-b)\sqrt{\frac{E}{\bar{T}}} + (a+b)\sqrt{\frac{\bar{T}}{E}} + \frac{2\delta b}{\bar{T}}\Big] \tag{6.30}$$

From this we can derive the following expression for $\frac{dp}{dE}$:

$$\frac{dp}{dE} = \frac{1}{4(1-\delta)}\left[\frac{\delta a A\bar{T}^b L^{1-a-b}}{1-\delta}\right]^{\frac{1}{1-a}}\left[\frac{1-a-b}{\sqrt{\bar{T}}}E^{-\frac{1}{2}} - \sqrt{\bar{T}}(a+b)E^{-\frac{3}{2}}\right] \tag{6.31}$$

And we can also derive the following expression for $\frac{p}{w}$, which shows that $\frac{p}{w}$ is again independent of A within this setup, which is likely to be again caused by the usage of log utility in consumption:

$$\frac{p}{w} = \frac{\delta a}{2(1-\delta)^2}\left[\sqrt{\frac{E}{\bar{T}}} + \frac{1}{1-a-b}\Big((a+b)\sqrt{\frac{\bar{T}}{E}} + \frac{2\delta b}{\bar{T}}\Big)\right] \tag{6.32}$$

Analysis of equation (6.31) shows the following condition for a positive relationship between E and p:

$$E > \bar{T}\frac{a+b}{1-a-b} \tag{6.33}$$

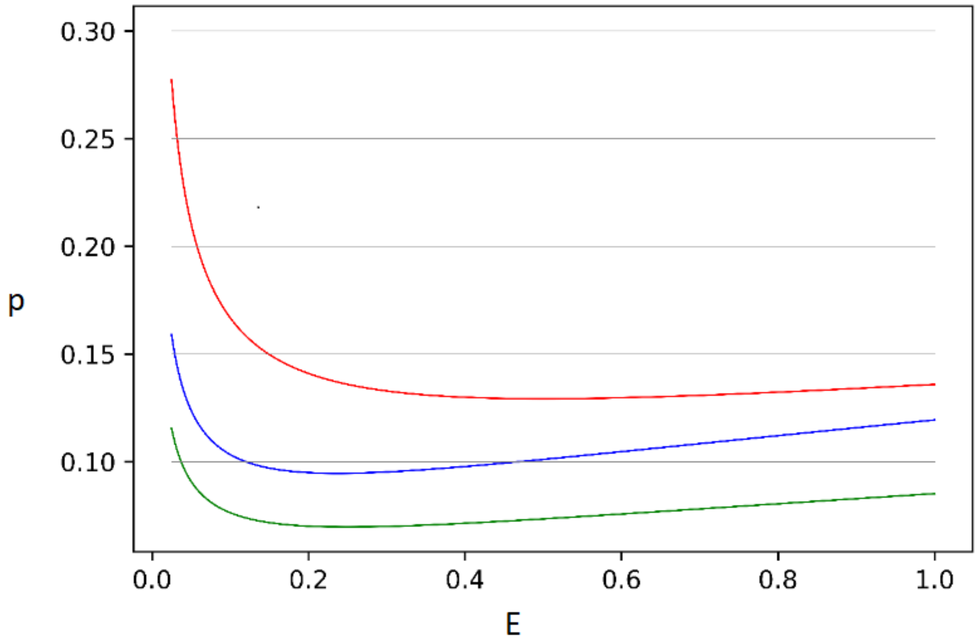
Which compares to the following equation from the static model:

$$E > \frac{a}{1-a} \tag{6.34}$$

W in the single asset model is equivalent to the combination of T and K in the two asset model. Thus the substitution of $a+b$, combined capital and land share of income, for just a, wealth share of income, is unsurprising, however it is interesting to find here that the size of $\bar{T}$ is now affecting the directionality of the relationship between p and E in a way that $\bar{W}$ did not in the single asset model. If total land is very small in this model, p will start to increase with E for very low levels of E. If total land is very large, p will likely stay decreasing in E for the whole range of E.

With the exception of the added importance of $\bar{T}$, the relationship between p and E is very similar to that of the static case, and that can be seen visually in 6.1 below.

Again, regardless of parameter values, with the single exception of $a = b = 0$, p tends to infinity as E tends to 0, and then aggressively decreases as E increases. p reaches a minimum at some leel of E, which is a fuction of a, b, and now also $\bar{T}$ and then slowly starts to increase. The value at which the minimum is reached is described exactly by equality of the inequality (6.33). At $\bar{T} = 1$, p will reach minima at identical levels of E to the static model when $a+b$ in the two asset model is equal to a in the static model.



Red: a=$\frac{1}{6}$ b=$\frac{1}{6}$
Green: a=$\frac{1}{10}$ b=$\frac{1}{10}$
Blue: a=$\frac{1}{6}$ b=$\frac{1}{40}$
For all lines L=$\bar{T}$=1 and δ=0.5

Figure 6.1: Graph of p against E in dynamic model steady state

So we can see that the relationship between p and E, and thus $\frac{p}{w}$ and E, has not changed fundamentally as a result of making the model dynamic and introducing accumulable capital. Whilst I have not performed the extension of the dynamic model into consistent utility functions between agents, as this would be quite involved mathematically and perhaps beyond the scope of an MPhil thesis, I believe that it is fair to assume that it would probably yield similar results to the extension of the static model - that being the introduction of strict monotonicity.

6.3 The dynamic model with utility from "net worth"

As discussed at the beginning of this chapter, it might be argued that the wealthy target some measure of "net worth", rather than specifically holdings of non-reproducible assets. Under such an assumption, it would be sensible for $K + pT$ to appear in the utility function of the rich rather than simply T. For this reason, I repeated the dynamic model using the the following utility function for the individual rich agent:

$$U = \sum_{t=0}^{\infty} \delta^t h(C_t, K_t + p_t T_t) \tag{6.35}$$

Using this utility function we reach the following two optimality conditions from the savings/spending decision of the rich agent:

$$h_1(C_t, K_t + p_t T_t) = h_2(C_t, K_t + p_t T_t) + \delta(1 + r_{t+1}) h_1(C_{t+1}, K_{t+1} + p_{t+1} T_{t+1}) \tag{6.36}$$

$$p_t h_2(C_t, K_t + p_t T_t) + \delta h_1(C_{t+1}, K_{t+1} + p_{t+1} T_{t+1}) = p_t h_1(C_t, K_t + p_t T_t) \tag{6.37}$$

Where again $h_x(a, b)$ signifies the derivative of the function h with respect to the xth argument at point (a, b).

6.3.1 Steady state analysis

At steady state the above conditions reduce to:

$$\frac{h_2(C, K + pT)}{h_1(C, K + pT)} = 1 - \delta(1 + r) \tag{6.38}$$

and

$$\frac{h_2(C, K + pT)}{h_1(C, K + pT)} = 1 - \delta(1 + \frac{\rho}{p}) \tag{6.39}$$

Indicating that p is fixed and equal to $\frac{\rho}{r}$ for all values of E. In this situation, where the consumption good is freely transferrable into reproducible capital, and reproducible capital is considered a direct substitute for non reproducible capital or "land" by savers, then land will inherit from reproducible capital the attribute of having no interesting price behaviour. In this case, we have returned to the classic Feldstein (1977) result for the price of assets as a function of marginal returns.

Chapter 7

The OLG model extension

7.1 Motivation - welfare implications

I have so far demonstrated that, under certain assumptions, wealth inequality can have interesting effects upon asset prices and interest rates. For sensible parameter values and assumptions, it would generally seem that high levels of inequality can push asset prices up and, correspondingly, push interest rates down.

However, it has generally been true throughout these models that the rising asset prices and falling interest rates have not directly affected the "welfare" of the poor agent in the sense that it has not generally affected his consumption or utility. In the majority of models, in which the poor has simply been a consumer of his wage income, his consumption has simply been a direct consequence of the wage, which, resulting directly from the production function which has been invariant in these models to changes in E, has not been changing in response to E in any way. As such, changes in E have not at all affected either the consumption or the utility of the poor agents in these models.

The only exception to this has been the extension to the static model in which both the poor and rich have the same utility function, caring explicitly about both consumption and wealth savings. Within this model, the price of the asset is monotonically decreasing in equality, whereas the wage of the poor is, as always in these models, invariant to equality. The way that equality is defined in this model is such that increases in equality do not at all increase the starting wealth allocation of the poor, which is always defined at zero. However, as equality increases, the price of the asset, in terms of the consumption good, falls. This will unambiguously improve the utility of the poor agent, who has a wage which is constant in terms of the consumption good, and derives explicit utility from purchases of the asset. In this model, the poor are made unambiguously better off by increases in equality, even though it does not change their starting position, simply because it leads to cheaper asset prices which improves their ability to purchase assets.

Whilst there is a case to be made that the poor do derive explicit utility from being able to purchase assets, many would argue that consumption is the key parameter for the poor, and that a utility loss that comes simply from a reduced ability to purchase assets is unimportant to the poor if it does not affect consumption. Whilst I suspect that not many non home-owning residents of big cities would make this argument, it is true that, within the confines of these models so far, increasing asset prices and decreasing interest rates do not prevent the poor from consuming in the same manner that they could have done before changes caused by decreasing equality.

This raises the question of why so many poor people are very concerned by increasing unaffordability of housing. One answer could be that, just like in my model where the poor, like the rich, gain explicit utility from owning assets, the poor simply like to be able to own assets. But there is also another mechanism whereby increasing asset prices, and decreasing interest rates hurt the poor. Even if the poor care only about consumption within their own lifetimes, they commonly follow a "life-cycle savings path", accumulating savings during the period they work. By doing this, the poor are able to hold assets within their lifetimes, which receive an income (or, often, reduce the expenses necessary to obtain housing services) during their lifetime, and also provide them an income in retirement. If assets are expensive, relative to wages, this will reduce the ability of workers to buy them and gain a share of the capital share of income. If the effective interest rates on these assets is low, then the return that the poor will be able to get for the assets that they do buy will be low, affording them a low income. The rich, on the other hand, will be benefited by increases in the asset price/consumption good ratio - it will enable them to obtain a greater amount of consumption if they were ever to choose to sell their assets.

I believe that this mechanism is an important part of the reason why poor people are often deeply concerned by rising asset prices, especially the prices of assets that provide essential services, such as housing. I am keen to explore whether this mechanism, when incorporated into this model, can provide an explanation of a way in which unaffordable assets directly impact the consumption levels and welfare of poor people.

7.2 The model

7.2.1 Assumptions

As has already been discussed, conventional capital accumulation equations forbid the ability of capital prices to behave in an interesting way. We have seen that, within a dynamic, two asset model, it is possible for prices of the non-reproducible asset to behave interestingly, provided that the rich derive explicit utility from holding them. However, within this context non-reproducible assets traded at a premium to reproducible capital due to their explicit utility effects for the rich. In such a model, poor people, if they were prioritising only consumption, would always have an incentive to use only reproducible capital for saving. As such, to explore the question of whether unaffordable assets can affect the lifetime consumption of the poor through hindering their ability to access assets, we must return to the model where all assets are affected uniformly by asset price changes, that being the single asset model. As such I will be returning to the single asset model, where W represents all assets and is fixed, for the entirety of this extension.

We will of course need to enable the poor to save, since this model focuses on how their ability to save is affected by changes in asset price due to inequality. The model will be dynamic and will incorporate three agents - a rich agent who is infinitely lives and a 2-generation poor agent who works and saves when young, and spends from his saved wealth when old. The rich will, consistent with models up til now, derive explicit utility from wealth, whereas the poor will be focused only on lifetime consumption. I owe this basic model concept to Michl (2007), in which a similar model of infinitely lived rich and two-generation poor is used. Statistics from the Federal Reserve Survey of Consumer Finances (US Federal Reserve, 2016) show that the vast majority of people leave very little or no inheritance, relative to their lifetime income, whereas the very rich tend to leave very large inheritances, so I believe it is a well justified assumption to make that the poor focus on lifetime consumption, whereas the rich behave "dynastically" as implied by this assumption.

7.2.2 Utility functions and budget constraints

I extend the utility function of the rich into multiple periods in the same fashion as I have done up until now:

$$U = \sum_{t=0}^{\infty} \delta_r^t f_r(C_{rt}, W_{rt}) \tag{7.1}$$

Where the r subscripts signify that the utility function and discount factors are those of the rich agent. I will not experiment with giving the agents different discount factors, but include this to show that it could theoretically be possible. I return to the use of W for capital/land/wealth to signify that I am again in a fixed asset world.

The budget constraint of the rich is:

$$W_{rt+1} = W_{rt}(1 + \frac{r_t}{p_t}) + \frac{w_t}{p_t} - \frac{C_{rt}}{P_t} \tag{7.2}$$

Where W_{rx} and C_{rx} are the wealth and consumption, respectively, of the rich agent in time x.

The poor, as described above, care only about consumption over a 2-period life and thus maximise:

$$f_p(C_{ypt}) + \delta_p f_p(C_{opt+1}) \tag{7.3}$$

With *ypt* and *opt* subscripts signifying the young poor and old poor respectively, both at time t.

The budget constraint of the poor is:

$$C_{opt+1} = \frac{w - C_{ypt}}{p_t}(p_{t+1} + r) \tag{7.4}$$

7.2.3 Solving the optimisations

The optimisation problem of the poor can be easily solved to give the following optimality condition:

$$f'_p(C_{ypt}) = \delta_p f'_p\Big(\frac{w - C_{ypt}}{p_t}(p_{t+1} + r)\Big)\frac{p_{t+1} + r}{p_t} \tag{7.5}$$

The optimisation problem of the rich is only slightly more difficult and gives the following solution:

$$\delta_r\Big[f_{r2}(C_{rt+1}, W_{rt+1}) + p_{t+1}f_{r1}(C_{rt+1}, W_{rt+1})\big(1 + \frac{r}{p_{t+1}})\Big] = p_t f_{r2}(C_{rt}, W_{rt}) \tag{7.6}$$

Where f_{rx} denotes the utility function of the rich differentiated with respect to the xth argument.

7.2.4 Substituting specific functions

Anticipating some problems with solvability with the most general functions at this point I substitute specific functions. For the rich I use the same specific function that I have used many times up until now:

$$f_r(C_{rt}, W_{rt}) = \ln C_{rt} + \sqrt{W_{rt}} \tag{7.7}$$

For the poor, log utility can somewhat simplify the mathematics of solving:

$$f_p(C_p) = \ln C_p \tag{7.8}$$

The optimal consumption of the rich is thus determined by the following equation:

$$\delta_r\Big[\frac{1}{2\sqrt{W_{rt+1}}} + \frac{p_{t+1}}{C_{rt+1}}\big(1 + \frac{r}{p_{t+1}}\big)\Big] = \frac{p_t}{C_{rt}} \tag{7.9}$$

The usage of log utility trivialises the optimality decision of the young poor which reduces simply to:

$$C_{ypt} = \frac{w}{1+\delta_p} \tag{7.10}$$

This allows us also to derive an exact expression for the savings of the old poor at any time $t+1$, in wealth units:

$$S_{opt+1} = \frac{\delta_p}{1+\delta_p}\frac{w}{p_t} \tag{7.11}$$

7.2.5 Solving for steady state

Solving for steady state by setting variables equal across time periods gives the following optimality condition for the individual rich:

$$C_r = \Big[\frac{1-\delta_r}{\delta_r}p - r\Big]2\sqrt{W_r^*} \tag{7.12}$$

Where W_r^* is here being introduced as the steady state wealth holding for each individual rich.

At steady state, W_r^* is constant across time, implying that:

$$C_r = W_r^* r + wL \tag{7.13}$$

We also know that the total wealth holdings of the rich, plus total wealth holdings of the old poor must equal the total wealth existing in the economy. Calling the total existing wealth $\bar{W}$ we then have:

$$\bar{W} = EW_r^* + (1-E)S_{op} \tag{7.14}$$

Substituting in equation (11) for S_{op} and rearranging we can thus reach the following expression for W_r^*:

$$W_r^* = \frac{\bar{W}}{E} - \frac{1-E}{E}\frac{w}{p}\frac{\delta_p}{1+\delta_p} \tag{7.15}$$

Setting the right hand sides of (12) and (13) equal, and subsituting this expression for W_r^*, we can thus obtain an expression for p in terms of only exogenous paramaters (recall that $\bar{W}$ and L are fixed and exogenous, so r and w are also exogenous).

$$r\Big(\frac{\bar{W}}{E} - \frac{1-E}{E}\frac{w}{p}\frac{\delta_p}{1+\delta_p}\Big) + wL = \Big[\frac{1-\delta_r}{\delta_r}p - r\Big]2\sqrt{\frac{\bar{W}}{E} - \frac{1-E}{E}\frac{w}{p}\frac{\delta_p}{1+\delta_p}} \tag{7.16}$$

Finally we can substitute in parameters for r and w using the following classic Cobb-Douglas conditions:

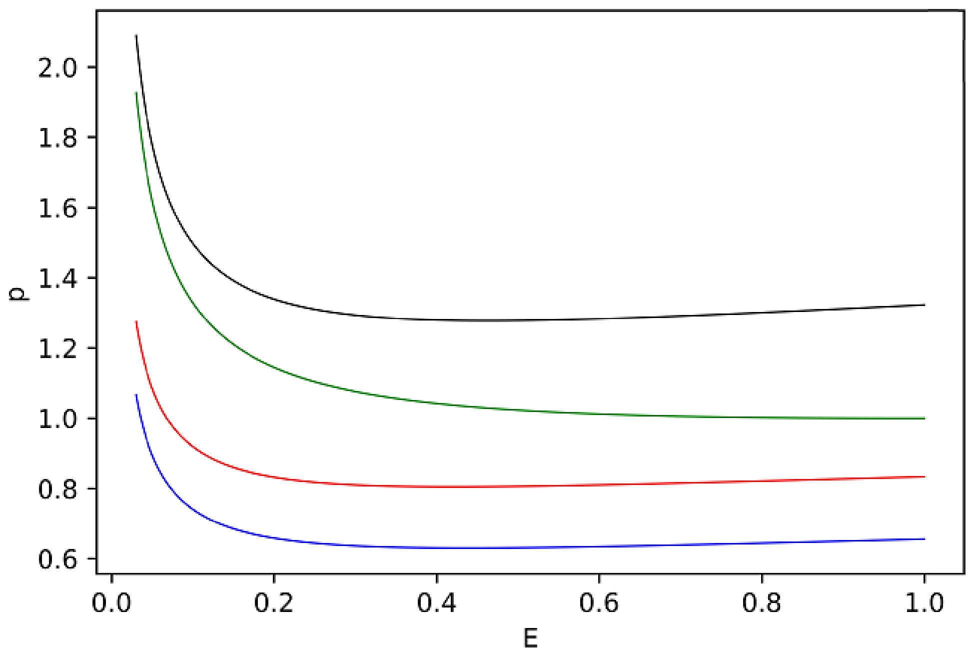
$$w = A(1-a)(\frac{\bar{W}}{L})^a \tag{7.17}$$

$$r = Aa(\frac{L}{\bar{W}})^{1-a} \tag{7.18}$$

Giving:

$$\begin{aligned} &Aa(\frac{L}{\bar{W}})^{1-a}\Big(\frac{\bar{W}}{E} - \frac{1-E}{E}\frac{A}{p}(1-a)\big(\frac{\bar{W}}{L}\big)^a\frac{\delta_p}{1+\delta_p}\Big) + A(1-a)(\frac{\bar{W}}{L})^a L = \\ &\Big[\frac{1-\delta_r}{\delta_r}p - Aa(\frac{L}{\bar{W}})^{1-a}\Big]2\sqrt{\frac{\bar{W}}{E} - \frac{1-E}{E}\frac{A}{p}(1-a)\big(\frac{\bar{W}}{L}\big)^a\frac{\delta_p}{1+\delta_p}} \end{aligned} \tag{7.19}$$

This is clearly somewhat unwieldy, and my attempts to simplify and solve algebraically were not successful. However, numerical solutions can be found for sensible suggestions of the variables $\bar{W}, L, A, a$ and the two δ's. I have plotted graphs of p against E for a variety of sensible parameter choices, and I include them in 7.1 below. The graphs show the relationship between p and E to be extremely similar to those in the base case, static model. To make this clearer, I have used the same set of a, $\bar{T}$ and L paramenters as in fig 4.1, and include again a copy of fig 4.1 for comparison. With the exception of some scaling, the two graphs appear to be identical. For the lines which achieve minima within the range $E \in (0, 1]$, those minima appear to occur at identical points.



Red: $a = \frac{1}{3}$ $\bar{W} = 1$ $L = 1$

Green: $a = \frac{1}{2}$ $\bar{W} = 1$ $L = 1$

Blue: $a = \frac{1}{3}$ $\bar{W} = 2$ $L = 1$

Black $a = \frac{1}{3}$ $\bar{W} = 1$ $L = 2$

For all lines $\delta_r = \delta_p = \frac{1}{2}$

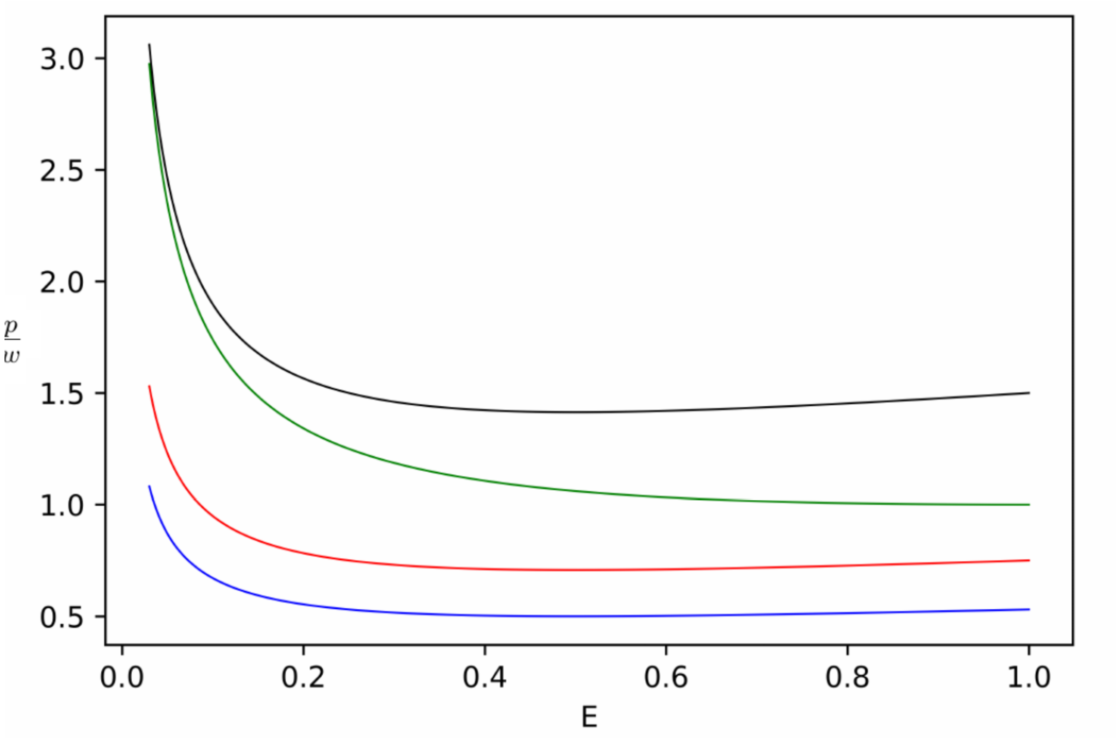
Figure 7.1: Graph of p against E in OLG model

7.2.6 Welfare implications

We know, from equation (7.10), that:

$$C_{yp} = \frac{w}{1 + \delta_p} \tag{7.20}$$

And we also know that:



Black: $a = \frac{1}{3}, \bar{W} = 1, L = 2$
Green: $a = \frac{1}{2}, \bar{W} = 1, L = 1$
Red: $a = \frac{1}{3}, \bar{W} = 1, L = 1$
Blue: $a = \frac{1}{3}, \bar{W} = 2, L = 1$

Figure 7.2: Repeat of fig 4.1

$$C_{op} = S_{op}(p + r) = \frac{\delta_p}{1 + \delta_p} \frac{w}{p}(p + r) = \frac{\delta_p w}{1 + \delta_p}(1 + \frac{r}{p}) \tag{7.21}$$

The right hand side of (7.20) is exogenous and fixed and the rhs of (7.21) is unambiguously decreasing in p. Thus the steady state consumption and welfare of the poor are unambiguously decreasing in steady state p. This is happening because the poor are less able to purchase assets when the price of them is high relative to wages, and are thus less able to receive a larger share of the capital income in their retirement. This effect is thus higher when the capital share of income is large. It is also larger when the poor have a large δ_p, as they have a larger desired savings in this case, and their decreased consumption is coming about through the decreased ability of the poor to save.

Chapter 8

Discussion and Further Work

8.1 Summary of results

The models put forward in this thesis show that, under the assumptions of fixed capital and diminishing marginal propensity to consume in total income, wealth distribution plays a key role in influencing asset prices and, thus, the effective interest rates of assets. The effect of inequality upon asset prices is shown to be negative for usual levels of inequality and capital share of income, and extremely negative for very high levels of inequality. If the assumption of diminishing marginal propensity to consume is extended from only rich agents to all agents, the relationship becomes unambiguous and monotonic. It is shown that technological progress can accentuate or diminish this effect, depending on the speed at which utility from consumption is satiated.

When the model is extended to the infinite time horizon, and reproducible capital is allowed as well as fixed capital, the same general relationship can be shown to hold between the wealth distribution and the price and interest rates of fixed capital as in the static, single asset model, but this is dependent upon the choice of utility function for the rich agent. When the rich agent gains explicit utility from holding the fixed capital, the relationship is maintained. If the rich agent instead targets a combination of the two assets, the relationship breaks down, and reverts to the same price relationship that always exists between consumption goods and reproducible capital under usual capital accumulation equations.

Finally the model demonstrates that, under an overlapping generations framework where the rich are infinitely lived and the poor live a two-period life where they work and save for retirement, the relationship between asset prices and inequality remains unaffected. Thus increases in inequality can negatively affect the ability of the poor to consume across a whole lifetime, by hampering their ability to access the capital share of income over the course of their lifetime and retirement.

8.2 Discussion of validity of results

The validity of the results depends, as always, on the validity of the assumptions. That marginal propensity to consume additional income is falling in lifetime income is well supported by a number of recent papers, which I have mentioned in my literature review (Straub (2018), Attanasio (1994), Dynan et al. (2004)).

The assumptions on capital are less innocuous. It is clear that the amount of capital in an economy varies, especially over the long and medium term, and, as such, the models within this thesis which are based upon completely fixed capital may seem, to some, to be unrealistic. In the instance of the model where both fixed and accumulable capital were allowed for, the results only held when agents targeted specifically fixed capital holdings, as opposed to "net value", which, again, some readers may find unrealistic.

I believe that more discussion of this particular assumption is needed. I do not believe it is true that capital is fixed. But I also do not believe it is true that capital can be formed effortlessly from consumption goods. Indeed, the past decade of global real interest rates planted firmly at, or below, zero, shows us that, in the real economy, situations can often exist where it is very difficult for savers to form new capital at all. Indeed, recent record high amounts of stock buybacks performed by large corporations imply that, rather than creating new capital, many firms are, in fact, focusing on returning capital to shareholders, even in the face of record low interest rates. In such an economy as this, and as an individual with personal experience of saving at negative real interest rates for some time, it seems to me sensible to assume that savers are unable to create new capital simply by saving. As such, I believe that these results do hold validity, especially in the current economic climate. I believe that more discussion of capital accumulation, its causes and especially its behaviour in the post-2008 zero interest rate economy is needed, and I would be very keen to read any such discussion.

Overall, I believe that the results are valid and interesting. But, as always in economics, we must be mindful of the assumptions used and whether they are accurate or relevant.

8.3 Significance of results and policy relevance

If the mechanism described in this work was believed to be a true cause of recent moves in asset prices and interest rates, it would be of quite some significance. For one thing, it would suggest that large increases in asset prices are not irrational or the result of a temporary "bubble" as is sometimes suggested, but are rather the natural state of an economy with high wealth inequality. Interest rates, also, which are constantly being predicted to raise back to "normal" historical levels, would be implied to actually be permanently low, due to new higher levels of wealth inequality, unless, for some reason, wealth inequality could be predicted to fall back down. This is in stark contrast to current market predictions which are, and have incorrectly been for the last 11 years, for a medium term normalisation of interest rates.

Beyond suggesting that these recent moves are permanent, the work also suggests that the causes for some of these moves may have been misattributed. A variety of reasons have been suggested for the broad rises in asset prices. Often these reasons differ depending on the particular asset being discussed. A variety of reasons have also been given for the unexpectedly prolonged fall in global interest rates. I have never heard increased wealth inequality discussed as the cause. If it was correct that wealth inequality was a significant cause in both these phemonena, it would both explain why both phenomena have been so unexpectedly persistent, and may also be important in knowing how to both understand and resolve these issues, particularly in cases where they have caused significant discontent, such as the increased unaffordability of housing in big cities. If high and growing wealth inequality are part of the reason for decreased housing affordability across a number of cities in a variety of countries, then reducing wealth inequality may be a key factor in resolving these problems. At the very least, wealth inequality should be more accurately measured so as to ascertain more clearly the role it is playing.

An important significance of this mechanism is the self-reinforcing effect it could have upon inequality itself. If high wealth inequality leads to expensive assets, then it does, in a sense, lead to higher wealth inequality, as higher asset prices relative to wages improve the economic positions of the wealthy relative to the poor. If this, in turn, makes it more difficult for poor people to save for retirement, as has been displayed in my OLG extension, then it is likely that people with relatively small wealth holdings may sell their wealth holdings to the wealthy in order to subsidise low retirement savings. By this mechanism wealth inequality could lead directly to greater wealth inequality, causing a dangerous self-reinforcing cycle which I believe we can see evidence of today by looking, for example, at the rapid fall in home ownership rate in the UK in the last 20 years.

A final significance is the effect that this mechanism could have on social mobility. When asset prices rise, relative to wages, it becomes difficult for working people with no inheritance to purchase assets. As this process becomes more extreme, it can be expected that distributions of wealth will become less related to work outcomes and more related to inheritances. At the very extreme, work outcomes could have almost no effect on the wealth distribution at all, moving society into a new feudalism, where relative social position is largely an inherited phenomenon. This is already relatively clear to people of my generation, where wealth ownership is largely determined by family, rather than work factors, and has even started to be noticed by economists, as shown by the recent work of Thomas Piketty. This is a fundamental challenge to the concept of fairness in capitalism. If increasing inequality leads to decreasing social mobility, then it could lead to a calcification of social classes over the long term.

With reference to the work of Thomas Piketty. Some criticism has been made of his work on recent increases of wealth inequality, most notably Rognlie (2014), that much of the increase in equality can be directly attributable to increases in house prices. This thesis posits the idea that increased house prices could themselves be a direct result of increased wealth inequality, somewhat countering that criticism. It also shows how those increased prices can, themselves, directly hurt the poor, countering the criticism further.

8.4 Further work

I believe that the basic framework I have devised of simple and easily varied inequality could be usefully extended. Questions could be asked, for example, about what happens if the rich obtain utility directly from withdrawing the fixed asset from productive use, rather than simply owning it. We could also allow labour supply to be a function of income, such that poorer people were more willing to provide labour to make up for their lower overall incomes. An extension that I am very interested in is the idea of allowing for two separate consumption good assets, one consumed by the poor and one by the rich, and seeing how an increase in wealth inequality affects the wage of the poor relative to their desired consumption good. Overall I think it is a rich and simple framework which could be used to approach a number of questions in a relatively mathematically simplistic way.

The OLG extension displays a dynamic which could be very interesting to explore. Using logarithmic utility for the poor, the poor choose to consume a fixed portion of their wage income in their first period. However, this does not hold generally for all utility functions. In general, the consumption of the poor in the first period will be a function of the price of assets relative to wages. If assets are cheap, their effective interest rate will be higher. Under many utility functions this will induce the poor to save more. Conversely, expensive assets yield lower effective interest rates, and could disincentivise the poor from saving – were dissaving possible, high enough asset prices could cause it to be chosen.

A more complex version of the model could be envisaged with three tiers of wealth holders – the rich, with large holdings, a middle class with small holdings, and a poor class with no holdings. Within this model we could seek to explore the feasibility of the possibility where an increase concentration of wealth within the rich pushes asset prices up and incentivises the middle class to dissave their assets. A model such as this could be useful in explaining recent decreases in home ownership in the UK.

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Postscript

Thanks for reading my thesis, I wrote it because I believe that the mechanism that it describes is a real thing that is happening in the real economy and causing harm to real people. I expect wealth inequality to continue to worsen, as the dynamic described in the model is self reinforcing. Thus, according to this theory, I expect asset price to wage ratios to continue to worse until a point where housing will never again be affordable to working people, and will become the privelige primarily of inherited wealth. I think that that will be a bad thing.

Thank you.

Chapter 9

Appendix

9.1 Proof of single, positive solution of p

We have the following equation:

$$C_r = 2p[(1 + (1 + \frac{r}{p})W_i + \frac{w}{p}L)^{\frac{1}{2}} - 1] \tag{9.1}$$

Taking $2p$ inside the bracket:

$$C_r = [4p^2 + (4p^2 + 4pr)W_i + 4pwL]^{\frac{1}{2}} - 2p \tag{9.2}$$

Make the assumption that there exists some p such that C_r is not increasing in p. Mathematically then, for that p we have:

$$\frac{dX}{dp} = \frac{1}{2}[4p^2 + (4p^2 + 4pr)W_i + 4pwL]^{-\frac{1}{2}}(8p + 8pW_i + 4rW_i + 4wL) \leq 2 \tag{9.3}$$

All parameters must be positive so we can rearrange to:

$$2p + 2pW_i + rW_i + wL \leq [4p^2 + (4p^2 + 4pr)W_i + 4pwL]^{\frac{1}{2}} \tag{9.4}$$

Again, both sides are clearly positive, so we can square both sides which leaves:

$$4p^2W_i^2 + 4prW_i^2 + 4pW_iwL + r^2W_i^2 + 2rW_iwL + w^2L^2 \leq 0 \tag{9.5}$$

Which is a contradiction since all terms on the LHS are strictly positive. Thus C_r is strictly increasing in p for all values of p

An identical procedure can be undertaken to show that C_p is also strictly increasing in p for all values of p.

Since we know that the following must hold:

$$Y = EC_r + (1 - E)C_p \tag{9.6}$$

And that Y is exogenous, it must then be that, if there is a solution for p for a given E within the range (0,1], then that solution is then unique, since the left hand side is constant and the right hand side is strictly increasing in p.

What remains, then, is to show that Y is within the range of the right hand side of the above equation. This will be proved if we can show that both C_r and C_p approach 0 as p goes to 0, and, since C_p is non negative, that C_r goes to infinity as p goes to infinity.

Equation (9.2) clearly shows that if p is 0, then C_r is 0, and the same can be shown for C_p. Thus there always exists a p sufficiently low that total consumption is lower than the exogenous equilibrium level.

We can also perform the following algebra to show that C_r is unbounded above as p increases:

$$\begin{aligned} &C_r = [4p^2 + (4p^2 + 4pr)W_i + 4pwL]^{\frac{1}{2}} - 2p \\ &C_r > [4p^2(1 + W_i)]^{\frac{1}{2}} - 2p \\ &[4p^2(1 + W_i)]^{\frac{1}{2}} - 2p = 2p(1 + W_i)^{\frac{1}{2}} - 2p = 2p[(1 + W_i)^{\frac{1}{2}} - 1] \end{aligned} \tag{9.7}$$

Which goes to infinity as p goes to infinity, since W_i is positive and $[(1+W_i)^{\frac{1}{2}} - 1]$ is a positive constant. C_p is strictly greater than a term which goes to infinity as p goes to infinity, thus C_p is unbounded above.

It is thus shown that total demand is strictly monotonically increasing from 0 to infinity as p increases from 0 to infinity. It thus achieves every positive value by the intermediate value theorem, and, due to the strict monotonicity, each value is achieved at a unique positive value of p.

9.2 Numerical solving

In order to acquire some of the diagrams in the thesis, I used numerical solving techniques as I had been unable to solve the equations with respect to p or $\frac{p}{w}$ analytically. To get p as a function of E for the diagrams, I write the equations in the form:

$$F(E, p) = 0 \tag{9.8}$$

Computationally, I then create a grid $\overline{E}$ of 10,000 points $\{E_i \in [0.03, 0.99999]\}$, and a grid of 50,000 points $\{p_i\}$ in appropriate intervals for each diagram. Then, for each point E_i, I evaluate the absolute value of F and find the p that minimises this; checking that it was sufficiently close to 0 (ie $F < \varepsilon = 0.0001$ This is appropriate because there was a unique minimiser over the range in each case. I then keep

$$p_i^* = \text{arg min } |F(p \mid E_i)| \tag{9.9}$$

for that level of E_i.

Plotting $\{p_i^*\}$ for each $\{E_i\}$ then gives me the diagrams.